MATH 323
QUIZ 9
NOVEMBER 19, 2009

NAME: SOLUTIONS

SHOW YOUR WORK FOR FULL CREDIT.

1. Show that if A^2 is the zero matrix, then the only eigenvalue of A is 0. Show all your work, do not skip any steps.

$Ax = \lambda x, \quad x \neq 0$

$A(Ax) = A(\lambda x), \quad x \neq 0$

$A^2 x = \lambda Ax, \quad x \neq 0$

$0x = \lambda \lambda x, \quad x \neq 0$

$0x = \lambda^2 x, \quad x \neq 0$

$\lambda^2 = 0$

$\lambda = 0$

2. Given that $\lambda = 4$ is an eigenvalue of $\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 1 \\ -3 & 4 & 5 \end{bmatrix}$, find one corresponding eigenvector.

$$\begin{bmatrix} -1 & 0 & -1 \\ 2 & -1 & 1 \\ -3 & 4 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & +1 & +1 \\ 0 & +4 & +4 \\ 0 & 0 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} -x_3 \\ -x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

EIGENVECTOR

3. Find the characteristic equation of $A = \begin{bmatrix} 5 & -2 & 7 & 1 \\ 0 & 3 & -4 & -2 \\ 0 & 0 & 5 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, then use this characteristic equation to find the eigenvalues of A.

$$0 = \det \begin{bmatrix} 5-\lambda & -2 & 7 & 1 \\ 0 & 3-\lambda & -4 & -2 \\ 0 & 0 & 5-\lambda & 3 \\ 0 & 0 & 0 & -\lambda \end{bmatrix} = (5-\lambda)^2 (3-\lambda)(-\lambda)$$

$\lambda_1 = 5$
$\lambda_2 = 5$
$\lambda_3 = 3$
$\lambda_4 = 0$

4. Show that if $A = QR$ with Q invertible, the A is similar to $A_1 = RQ$.

$A_1 = RQ = Q^{-1}QRQ = Q^{-1}(QR)Q = Q^{-1}AQ$

So, A_1 IS SIMILAR TO A.

5. Use the properties of determinants to show that A and A^T have the same eigenvalues.

$0 = \det(A^T - \lambda I) = \det((A - \lambda I)^T) = \det(A - \lambda I)$.

A AND A^T HAVE THE SAME CHARACTERISTIC EQUATION $\Rightarrow$ SAME EIGENVALUES.