MATH 323
QUIZ 8
NOVEMBER 12, 2009

NAME: SOLUTIONS

SHOW YOUR WORK FOR FULL CREDIT.

1. Let $\mathcal{B} = \left\{ \begin{bmatrix} -5 \\ 3 \end{bmatrix}, \begin{bmatrix} 6 \\ -4 \end{bmatrix} \right\}$ and $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$. Find the vector $\mathbf{x}$.

$$x = 3\begin{bmatrix} -5 \\ 3 \end{bmatrix} + 5\begin{bmatrix} 6 \\ -4 \end{bmatrix} = \begin{bmatrix} 15 \\ -11 \end{bmatrix}$$

2. The set $\mathcal{B} = \{1+t, 1+t^2, t+t^2\}$ is a basis for $\mathbf{P}_2$. Find the coordinate vector of $\mathbf{p}(t) = 6 + 3t - t^2$ relative to $\mathcal{B}$.

$$6+3t-t^2 = c_1(1+t) + c_2(1+t^2) + c_3(t+t^2)$$

$$\left.\begin{array}{l} 6 = c_1 + c_2 \\ 3 = c_1 + c_3 \\ -1 = c_2 + c_3 \end{array}\right\} \Rightarrow \begin{array}{l} c_1 = 5 \\ c_2 = 1 \\ c_3 = -2 \end{array}$$

$$[6+3t-t^2]_{\mathcal{B}} = \begin{bmatrix} 5 \\ 1 \\ -2 \end{bmatrix}$$

3. The first four Laguerre polynomials are $1$, $1-t$, $2-4t+t^2$, and $6-18t+9t^2-t^3$. Use the coordinate vectors of these polynomials, with respect to the standard basis $\{1, t, t^2, t^3\}$, to prove that the first four Laguerre polynomials form a basis for $\mathbf{P}_3$.

$$[1]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad [1-t]_{\mathcal{B}} = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

$$[2-4t+t^2]_{\mathcal{B}} = \begin{bmatrix} 2 \\ -4 \\ 1 \\ 0 \end{bmatrix} \quad [6-18t+9t^2-t^3]_{\mathcal{B}} = \begin{bmatrix} 6 \\ -18 \\ 9 \\ -1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 2 & 6 \\ 0 & -1 & -4 & -18 \\ 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$ HAS A PIVOT IN EVERY COLUMN

$\Rightarrow$ COLUMNS OF $A$ FORM A BASIS FOR $\mathbb{R}^4$

$\Rightarrow$ FIRST FOUR LAGUERRE POLY'S A BASIS FOR $\mathbb{P}_3$.

4. Suppose a $5 \times 6$ matrix $A$ has four pivot columns. What is the dimension of Nul $A$? Is Col $A = \mathbf{R}^4$? Justify your answers.

dim Nul $A = 2$ (= # OF FREE VARIABLES)

Col $A \neq \mathbb{R}^4$ SINCE Col $A$ IS A SUBSPACE OF $\mathbb{R}^5$.

5. Suppose $A = \begin{bmatrix} 2 & -1 & 1 & -6 & 8 \\ 1 & -2 & -4 & 3 & -2 \\ -7 & 8 & 10 & 3 & -10 \\ 4 & -5 & -7 & 0 & 4 \end{bmatrix}$ is row equivalent to $B = \begin{bmatrix} 1 & -2 & -4 & 3 & -2 \\ 0 & 1 & 3 & -4 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.
Find rank $A$, dim Nul $A$ and a basis for Row $A$.

rank $A = 2$. (= dim Col $A$)

dim Nul $A = 3$ (rank $A$ + dim Nul $A$ = 5)

$\mathcal{B}_{\text{Row}A} = \{(1,-2,-4,3,-2), (0,1,3,-4,4)\}$