MATH 323 NAME: SOLUTIONS

QUIZ 7

OCTOBER 29, 2009 SHOW YOUR WORK FOR FULL CREDIT.

1. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix}$, $\mathbf{v}_1 = \begin{bmatrix} 6 \\ 2 \\ -1 \end{bmatrix}$, $\mathbf{v}_1 = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$, $\mathbf{v}_1 = \begin{bmatrix} -4 \\ -8 \\ 9 \end{bmatrix}$. Find a basis for the subspace $W$ spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$.

$$\begin{bmatrix} 1 & 6 & 2 & -4 \\ -3 & 2 & -2 & -8 \\ 4 & -1 & 3 & 9 \end{bmatrix} \sim \begin{array}{l} \\ 3R_1+R_2 \\ -4R_1+R_3 \end{array}\begin{bmatrix} 1 & 6 & 2 & -4 \\ 0 & 20 & 4 & -20 \\ 0 & -25 & -5 & 25 \end{bmatrix} \sim \begin{array}{l} \\ \frac{1}{4}R_2 \\ \frac{1}{5}R_3 \end{array}\begin{bmatrix} 1 & 6 & 2 & -4 \\ 0 & 5 & 1 & -5 \\ 0 & -5 & 1 & 5 \end{bmatrix} \sim \begin{array}{l} \\ \\ R_2+R_3 \end{array}\begin{bmatrix} 1 & 6 & 2 & -4 \\ 0 & 5 & 1 & -5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathcal{B}_{\mathrm{Col}A} = \left\{ \begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix}, \begin{bmatrix} 6 \\ 2 \\ -1 \end{bmatrix} \right\}$$

2. Find a basis for Nul $A$ where $A$ is row equivalent to the matrix $\begin{bmatrix} 1 & 0 & 6 & -8 & 1 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

$x_1 = -6x_3 + 8x_4 - x_5$

$x_2 = 2x_3 - x_4$

$x_3$ free

$x_4$ free

$x_5$ free

$$\mathcal{B}_{\mathrm{Nul}A} = \left\{ \begin{bmatrix} -6 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 8 \\ -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

3. True or False. If $H = \mathrm{Span}\{\mathbf{b}_1, \ldots, \mathbf{b}_p\}$, then $\{\mathbf{b}_1, \ldots, \mathbf{b}_p\}$ is a basis for $H$. Justify your answer.

FALSE, $\{b_1, \ldots, b_p\}$ MIGHT BE LINEARLY DEPENDENT.

4. True or False. If $\{\mathbf{b}_1, \ldots, \mathbf{b}_p\}$ is a linearly independent set in a subspace $H$, then $\{\mathbf{b}_1, \ldots, \mathbf{b}_p\}$ is a basis for $H$. Justify your answer.

FALSE, $\{b_1, \ldots, b_p\}$ MIGHT NOT SPAN $H$.

5. Let $T : V \to W$ be a linear transformation from a vector space $V$ into a vector space $W$. Prove that the range of $T$ is a subspace of $W$. *Hint:* Typical elements of the range have the form $T(\mathbf{x})$ and $T(\mathbf{y})$ for some $\mathbf{x}$ and $\mathbf{y}$ in $V$.

(1) $0 \in V$, SO $T(0) = 0 \in$ RANGE$(T)$

(2) IF $T(x), T(y) \in$ RANGE$(T)$, THEN $T(x) + T(y) = T(x+y) \in$ RANGE$(T)$

(3) IF $T(x) \in$ RANGE$(T)$, $c$ ANY SCALAR, THEN
$cT(x) = T(cx) \in$ RANGE$(T)$.