MATH 323 NAME: Solutions

QUIZ 6

OCTOBER 22, 2009 SHOW YOUR WORK FOR FULL CREDIT.

1. Find a formula for $\det(kA)$ where A is an $n \times n$ matrix.

   SINCE EACH ROW IS SCALED BY A FACTOR OF k,

   $$\det(kA) = k^n \det A$$

2. Prove that if A and B are square matrices, then $\det AB = \det BA$.

   $$\det AB = \det A \det B = \det B \det A = \det BA$$

3. Use Cramer's rule to solve the linear system $\begin{array}{r} 3x_1 - 2x_2 = 6 \\ -5x_1 + 4x_2 = 8 \end{array}$.

   $$x_1 = \frac{\begin{vmatrix} 6 & -2 \\ 8 & 4 \end{vmatrix}}{\begin{vmatrix} 3 & -2 \\ -5 & 4 \end{vmatrix}} = \frac{24+16}{12-10} = \frac{40}{2} = 20$$

   $$x_2 = \frac{\begin{vmatrix} 3 & 6 \\ -5 & 8 \end{vmatrix}}{2} = \frac{24+30}{2} = \frac{54}{2} = 17$$

4. Let H be the set of all polynomials of degree 1, with integers as coefficients. Show that H is not a subspace of $\mathbf{P}_2$.

   $p(t) = t \in H$, BUT $\frac{1}{2}p(t) = \frac{1}{2}t \notin H$

5. Let F be a fixed $m \times n$ matrix, and let H be the set of all matrices A in $M_{n\times p}$ with the property that $FA = 0$ (the zero matrix in $M_{m\times p}$). Show that H is a subspace of $M_{n\times p}$.

   (a) $F0 = 0 \Rightarrow 0 \in H$

   (b) $A_1, A_2 \in H$, THEN $F(A_1+A_2) = FA_1 + FA_2 = 0 + 0 = 0 \Rightarrow A_1 + A_2 \in H$

   (c) $A \in H$, c ANY SCALAR, THEN $F(cA) = cFA = c0 = 0 \Rightarrow cA \in H$