MATH 323
QUIZ 5
OCTOBER 15, 2009

NAME: SOLUTIONS

SHOW YOUR WORK FOR FULL CREDIT.

1. Show that if the columns of A form a linearly independent set, the the equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution. You may not quote the Invertible Matrix Theorem.

IF THE COLUMNS OF A FORM A LIN. INDEP. SET THEN THE EQUATION $x_1 a_1 + \cdots + x_m a_m = 0$ HAS ONLY THE TRIVIAL SOLUTION. THEREFORE, THE EQUATION $Ax = 0$ HAS ONLY THE TRIVIAL SOLUTION.

2. If $n \times n$ matrices E and F have the property that $EF = I$, show that $FE = I$.

IF $EF = I$, THE (BY THE I.M.T.) $F = E^{-1}$.

THEREFORE,

$$FE = E^{-1}E = I$$

3. Show that the linear transformation from $\mathbf{R}^2$ into $\mathbf{R}^2$ defined by $T(x_1, x_2) = (5x_1 - 9x_2, -4x_1 + 7x_2)$ is invertible and find a formula for T^{-1}.

$T(x) = Ax$ WHERE $A = \begin{bmatrix} 5 & -9 \\ -4 & 7 \end{bmatrix}$. SINCE THE COLUMNS OF A ARE LIN. INDEP. (THEY ARE NOT SCALAR MULTIPLES OF EACH OTHER) BY THE I.M.T., T IS INVERTIBLE.

MOREOVER, $T^{-1}(x) = A^{-1}x$ WHERE

$$A^{-1} = \frac{1}{(5)(7) - (-9)(-4)} \begin{bmatrix} 7 & 9 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} -7 & -9 \\ -4 & -5 \end{bmatrix}$$

4. Compute $\det \begin{bmatrix} 5 & -7 & 2 & 2 \\ 0 & 3 & 0 & -4 \\ -5 & -8 & 0 & 3 \\ 0 & 5 & 0 & -6 \end{bmatrix}$ using the least amount of computation.

$$2 \det \begin{bmatrix} 0 & 3 & -4 \\ -5 & -8 & 3 \\ 0 & 5 & -6 \end{bmatrix} = 2(-1)(-5) \det \begin{bmatrix} 3 & -4 \\ 5 & -6 \end{bmatrix}$$

$$= 10 \left[(3)(-6) - (-4)(5) \right]$$

$$= 10(2)$$

$$= 20$$

5. Compute $\det \begin{bmatrix} 4 & 0 & 0 & 0 \\ 7 & -1 & 0 & 0 \\ 2 & 6 & 3 & 0 \\ 5 & -8 & 4 & -3 \end{bmatrix}$ by using the least amount of computation.

SINCE THE MATRIX IS TRIANGULAR WE HAVE

$(4)(-1)(3)(-3) = 36$