MATH 323 NAME: SOLUTIONS
QUIZ 4
OCTOBER 8, 2009 SHOW YOUR WORK FOR FULL CREDIT.

1. If $A = \begin{bmatrix} 3 & 2 \\ 7 & 4 \end{bmatrix}$, find A^{-1}.

$$A^{-1} = \frac{1}{(3)(4)-(2)(7)} \begin{bmatrix} 4 & -2 \\ -7 & 3 \end{bmatrix} = \frac{1}{-2} \begin{bmatrix} 4 & -2 \\ -7 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 7/2 & -3/2 \end{bmatrix}$$

2. True or False. If A and B are invertible, then $A^{-1}B^{-1}$ is the inverse of AB. Justify your answer.

   False, $(AB)^{-1} = B^{-1}A^{-1}$.

3. Prove that if A and B are invertible $n \times n$ matrices, then AB is also invertible.

   If A and B are invertible, then $(AB)^{-1} = B^{-1}A^{-1}$ exists.

4. Suppose A is an invertible $n \times n$ matrix. Show that if $\mathbf{b}$ is any element of $\mathbf{R}^n$, then the equation $A\mathbf{x} = \mathbf{b}$ is consistent.

   Let $\mathbf{x} = A^{-1}\mathbf{b}$, then $A\mathbf{x} = A(A^{-1}\mathbf{b}) = (AA^{-1})\mathbf{b} = I\mathbf{b} = \mathbf{b}$; therefore, $\mathbf{x} = A^{-1}\mathbf{b}$ is a solution of $A\mathbf{x} = \mathbf{b}$.

5. Prove that if $A = [\mathbf{a}_1 \ \dots \ \mathbf{a}_n]$ is invertible, then the columns of A are linearly independent.

   Since A is invertible (by Theorem 5, but not by problem 4 (why not?)), the equation $A\mathbf{x} = \mathbf{0}$ has the *unique* solution $\mathbf{x} = A^{-1}\mathbf{0} = \mathbf{0}$. Therefore, $x_1\mathbf{a}_1 + \cdots + x_n\mathbf{a}_n = \mathbf{0}$ has only the trivial solution showing that the columns of A are linearly independent.