Math 323 Name: Solutions

Quiz 2

September 17, 2009 Show your work for full credit.

1. Do the columns of $A = \begin{bmatrix} 1 & -3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ span $\mathbf{R}^4$? Justify your answer.

   Since A does not have a pivot position in each row, by Theorem 4, the columns of A do not span $\mathbf{R}^4$.

2. True or False? Justify your answer.

   Every matrix equation $A\mathbf{x} = \mathbf{b}$ corresponds to a vector equation with the same solution set.

   True. The matrix equation $A\mathbf{x} = \mathbf{b}$ corresponds to the vector equation $x_1\mathbf{a_1} + \cdots + x_n\mathbf{a_n} = \mathbf{b}$ where $\mathbf{a_1}, \cdots, \mathbf{a_n}$ are the columns of A.

3. Describe all solutions of $A\mathbf{x} = \mathbf{0}$ in parametric form where A is row equivalent to the matrix $\begin{bmatrix} 1 & 4 & 0 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

$$\begin{aligned} x_1 &= -4x_2 - 5x_6 \\ x_2 & \quad \text{free} \\ x_3 &= -x_6 \\ x_4 & \quad \text{free} \\ x_5 &= -4x_6 \\ x_6 & \quad \text{free} \end{aligned}$$

$$\mathbf{x} = x_2 \begin{bmatrix} -4 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_6 \begin{bmatrix} -5 \\ 0 \\ -1 \\ 0 \\ -4 \\ 1 \end{bmatrix}$$

4. Suppose $\mathbf{p}$ is a solution of $A\mathbf{x} = \mathbf{b}$ and $\mathbf{q}$ is a solution of $A\mathbf{x} = \mathbf{0}$. Show that $\mathbf{w} = \mathbf{p} + \mathbf{q}$ is a solution of $A\mathbf{x} = \mathbf{b}$. Do not skip any steps.

$$\begin{aligned} A\mathbf{w} &= A(\mathbf{p} + \mathbf{q}) \\ &= A\mathbf{p} + A\mathbf{q} \\ &= \mathbf{b} + \mathbf{0} \\ &= \mathbf{b} \end{aligned}$$