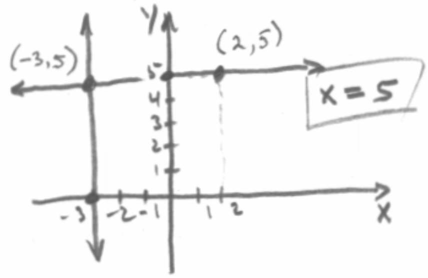
MATH 253
EXAM 1
OCTOBER 7, 2009

NAME: SOLUTIONS

SHOW YOUR WORK FOR FULL CREDIT.

1. Write the slope-intercept form of the equation of the line through the point $(2, 5)$ and perpendicular to the line $x = -3$.



2. Let $f(x) = \sqrt{x}$ and $g(x) = x^2 + 4$.

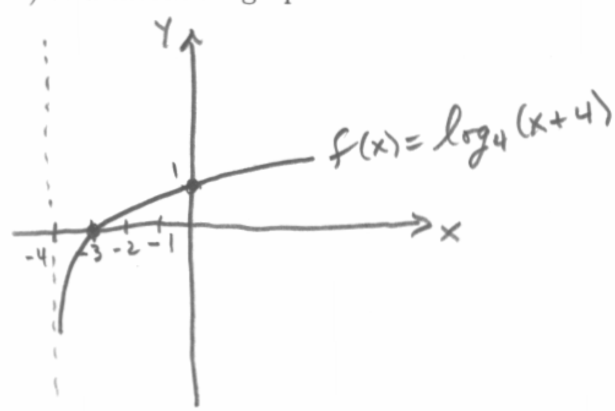
   (a) Find $(f \circ g)(x)$ and give its domain.

   $f(g(x)) = f(x^2+4) = \sqrt{x^2+4}$, $(-\infty, \infty)$

   (b) Find $(g \circ f)(x)$ and give its domain.

   $g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^2 + 4 = x + 4$, $[0, \infty)$

3. Find the domain, x-intercept, y-intercept, and vertical asymptote of the function $f(x) = \log_4(x + 4)$ and sketch its graph.



4. Expand the expression $\log \frac{\sqrt{5x-3}}{7}$. Simplify your answer.

$\log \sqrt{5x-3} - \log 7$

$\log (5x-3)^{1/2} - \log 7$

$\boxed{\frac{1}{2} \log (5x-3) - \log 7}$

5. Condense the expression $\frac{1}{3}[\ln x - \ln(x+1)]$ to the logarithm of a single quantity. Simplify your answer.

$\frac{1}{3} \ln \frac{x}{x+1}$

$\ln \left(\frac{x}{x+1}\right)^{1/3}$

$\boxed{\ln \sqrt[3]{\frac{x}{x+1}}}$

6. Solve the exponential equation $3(2^x) = 42$. Give the exact answer using *natural logarithms.*

$2^x = 14$

$\ln 2^x = \ln 14$

$x \ln 2 = \ln 14$

$\boxed{x = \frac{\ln 14}{\ln 2}}$

7. Solve the logarithmic equation $\log 5x + \log(x-1) = 2$. Give the exact answer(s).

$$\log 5x(x-1) = 2$$
$$5x(x-1) = 10^2$$
$$5x(x-1) = 100$$
$$x(x-1) = 20$$
$$x^2 - x - 20 = 0$$
$$(x-5)(x+4) = 0$$

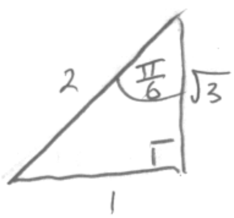
$\boxed{x = 5}$, $\cancel{x = -4}$ DOES NOT CHECK

8. Find the radian measure of the central angle of a circle of radius $r = 14$ feet that intercepts an arc of length $s = 8$ feet. Give the exact answer and simplify.

$$s = r\theta$$
$$8 = 14\theta$$
$$\theta = \frac{8}{14}$$

$\boxed{\theta = \frac{4}{7}}$

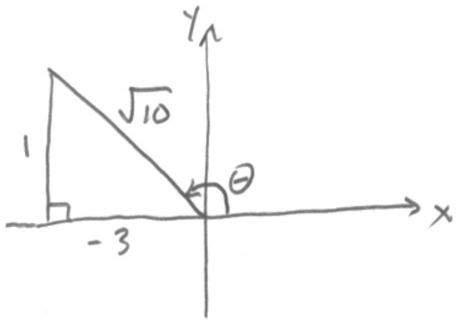
9. Find the exact value of θ in radians if $\csc\theta = 2$ and $0 \le \theta < \pi/2$.



$$\frac{1}{\sin\theta} = 2$$
$$\sin\theta = \frac{1}{2}$$

$\boxed{\theta = \frac{\pi}{6}}$

10. If $\cot\theta = -3$ and θ lies in Quadrant II, find $\sin\theta$.



$$\cot\theta = -3$$
$$\tan\theta = -\frac{1}{3}$$

$\boxed{\sin\theta = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}}$