

Departmental Syllabus **Math 3130 -- College Geometry**

Textbook: None (In the future, a notes packet will be available for students to purchase).

Prerequisites: MATH 2640 and junior standing or consent of department chair

Calculators: No specific calculator required.

Software: Cabri Geometry II will be used extensively in the course.

Course Description: Topics from Euclidean geometry including classical theorems, transformational geometry, and Euclidean constructions. Noneuclidean topics include inversion and reciprocation, as well as some ideas from projective geometry. A dynamic geometry software program is used extensively to illustrate ideas in the course

Topics to be covered:

A. Euclidean Geometry of Polygon and Circle (with reference to Projective geometry):

1. Directed ratios and products. Harmonic range.
2. Ideal points and the ideal line.
3. Concurrent lines and collinear points.
 - a. Perpendicular bisectors; the circumcenter and circumcircle of a triangle.
 - b. Theorem of Menelaus and its converse.
 - c. Theorem of Ceva and its converse.
 - d. Medians; the centroid of a triangle; the medial triangle of a triangle.
 - e. Internal and external angle bisectors; the incenter and incircle of a triangle; the excenters and excircles of a triangle.
 - f. Altitudes; the orthocenter of a triangle; the orthic triangle of a triangle.
 - g. Antiparallel segment in a triangle.
4. Circles
 - a. Inscribed and central angles in a circle.
 - b. Cyclic quadrilaterals.
 - c. The Power of a Point with respect to a circle.
 - d. A Simpson line of a triangle.
 - e. A Miquel point of a triangle.
 - f. The Gergonne and Nagel points of a triangle.
5. The nine point circle and 9-point center of a triangle.
6. The Euler line and Euler triangle of a triangle.
7. Apollonian Circles.
8. Theorems of Desargus, Pappus and Pascal.

B. Euclidean constructions and constructibility.

1. The collapsing compass.
2. Arithmetic constructions; division by n , multiplication, division and square root.

3. Constructible numbers; minimal polynomial over the rationals.
4. Constructible angles and polygons; Gauss's Theorem.
5. Trisection of an angle.
6. Euclidean constructions; construction of a triangle or circle from given information.

C. Isometries of the plane.

1. Introduction, algebraic discussion of isometries; even and odd isometries; fixed points and lines.
2. Description of the translation T_{AB} , the rotation $R(O,\alpha)$, the reflection \mathfrak{R}_l and the glide reflection G_{AB} .
3. Identifying an isometry given two congruent triangles.
4. Products of isometries; simplifying products; products of reflections; products of half-turns and translations.
5. Applications of isometries; using isometries in constructions.
6. Wallpaper patterns and tilings of the plane.

D. Similarities of the plane and homotheties (dilations).

1. Introduction, algebraic discussion of similarities; even and odd isometries; fixed points and lines.
2. Description of the homothety (or dilation) $H(O,r)$.
3. Identifying a homothety given two similar triangles with corresponding sides parallel.
4. Products of homotheties and translations; simplifying products.
5. Applications of homotheties; using homotheties in constructions

E. Inversion and the inversive plane

1. Description of the inversion $I(O,r)$.
2. The ideal point and the inversive plane; pole and polars.
3. Fixed points, lines and circles (orthogonal circles); the image of a circle or line.
4. Theorem's of Ptolemy and Feuerbach.
5. Use of Inversion in constructions.

F. Reciprocation.

1. Description of reciprocation with respect to the circle.
2. Conjugate points and lines; ideal points; the ideal line and the projective plane.
3. Duality and the dual configuration; complete quadrangle and quadrilateral
4. Self-polar triangle and the polar circle.
5. Reciprocal conic of a circle.
6. Theorems of Brianchon and Pascal.
7. Sterographic and gnomonic projections.