



PROBLEM OF THE WEEK #6
(Fall 2012)

Joe is currently a political prisoner in the totalitarian country of Cobra Island. He was given a very unusual sentence:

1. He will be released as soon as he reaches the final step of a 100-step stairway.
2. He will begin his sentence at the bottom, however he cannot climb at his own will: he is allowed to move only one step per day.
3. He can go up one step each day on those months that are odd, but he is required to go down one step each day on those months that are even. (Note: If we number the months as 1=Jan., 2=Feb., 3=Mar., 4=Apr. etc. the odd months are Jan., Mar., etc. and the even months are Feb., Apr., etc.).

His sentence began on January 1, 2001. On what exact date is Joe going to be released?

Solution:

We can see that during the first year of his sentence (the year 2001) the prisoner was moving between steps 1 and 36 of the stairs. This latter step, number 36, was reached on July 31. On December 31 of that year, he was back to step 3. In general, if on a December 31 he is on step n , then the next year he will either

- (a). move between steps $n + 1$ and $n + 36$ and will end the year on the step $n + 3$, if that next year *is not* a leap year,

or

- (b). move between steps $n + 1$ and $n + 35$ and will end the year on the step $n + 2$, if that next year *is* a leap year.

We can see, after some simple calculations, that on December 31, 2024 he will reach step 66 of the stairs, after having been on step 99 on July 31 of the same year as his highest point. From here we can see what will happen in the year 2025: on January 31 he will reach step 97, on February 28 he will be back to step 69, and on March 31, at last, he will reach step 100. Hence, he will be released on March 31, 2025.