MODELING AND ANALYSIS OF
FRICtIONAL COUPLING MECHANISM
IN MULTIDISK STEPLESS TRANSMISSION

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ABSTRACT

This paper presents an analysis of the frictional coupling mechanism in rolling variators with initial point contact. Rolling variators are continuously variable transmissions where the power is transmitted by frictional contact between non-flexible rotating elements. The essential feature of this type of transmission is that the adjoining surfaces rotate when the rolling elements roll against each other resulting in drilling-type friction due to drilling-type motion. The performance characteristics of rolling variators are thus evaluated on the basis of both the drilling-type friction and tangential forces, which transmit the power. Relationships between the characteristic parameters of frictional coupling elements are derived and expressed in the form of integrals, which can be readily evaluated numerically for the tangential force utilization coefficient, power loss coefficient, and geometric efficiency. The analysis of the frictional coupling mechanism in rolling variators constitutes a tangible contribution to the theory dealing with the design of this type of continuously variable transmissions.
# TABLE OF CONTENTS

1. INTRODUCTION ............................................................................................................ 3

2. BASIC KINEMATIC RELATIONS .................................................................................. 7
   2.1. Transmission Ratio in Unloaded Condition ............................................................. 10
   2.2. Transmission Ratio in Loaded Condition ................................................................. 10
   2.3. Relative Slip ........................................................................................................... 11
   2.4. Drilling Velocity .................................................................................................... 11

3. THE FRICTIONAL COUPLING MECHANISM IN MULTI-DISK STEPLESS TRANSMISSION .................................................................................................................. 12
   3.1. Tangential Forces Generated on the Contacting Surfaces of Rolling Elements with Initial Point Contact. ........................................................................................................... 12
   3.2. Tangential Force Utilization Coefficient .................................................................. 16
   3.3. Power Loss Coefficient .......................................................................................... 17
   3.4. Moment Arm of the Resultant Tangential Force ..................................................... 18
   3.5. Geometric Efficiency of the Frictional Pair .......................................................... 19

4. SUMMARY AND CONCLUSIONS ............................................................................... 19
   4.1. Summary of Numerical Results .............................................................................. 19
   4.2. Conclusions ............................................................................................................ 24

REFERENCES .................................................................................................................. 25

NOMENCLATURE .............................................................................................................. 25
1 INTRODUCTION

Common among various mechanisms used for stepless speed control are frictional transmissions, also known as mechanical continuously variable transmissions or rolling variators.

Frictional transmissions are mechanisms, which transmit motion and power from the driving shaft to the driven shaft by means of frictional forces between rolling elements due to pressure exerted on the elements’ surfaces of contact. The frictional forces are tangent to the contact surfaces and act in the circumferential direction.

A particular feature of these transmissions is that, in each pair of the contact surfaces, the surfaces rotate relative to each other about an axis normal to the contact area while the rolling elements perform their rolling motion. This motion of rotation is similar to that performed by a drill tip relative to the surface being drilled. Friction generated by this motion may be called drilling friction. It is because of this particular feature that the resulting drilling friction and the circumferential frictional force constitute a basis for evaluation of frictional transmissions.

Generally, frictional transmissions can be divided into two categories:

1. Transmissions with power transmitted from an active element (a) to a passive element (b) by means of an intermediate element (c) as shown in Fig. 1 (a). The transmission ratio is changed by changing the location of the intermediate element relative to the axis of rotation of the active and passive elements. Application of two or more intermediate elements loaded in parallel fashion protects against high contact pressures.

2. Transmissions without intermediate elements in which power is transmitted directly from an active element (a) to a passive element (b) as shown in Fig. 1 (b). The transmission ratio is changed here through change in location of the active element with respect to the passive element.
The transmissions mentioned above usually operate under dry friction conditions. Frequently, they have to meet two contradictory design requirements: both the circumferential frictional force and the life cycle of the frictional coupling elements should be as great as possible.

To meet the first requirement, it is necessary to generate a high coefficient of friction and, for the second, a low coefficient is indispensable. In case of steel frictional coupling elements, the coefficient of friction can reach considerable values (0.6 to 0.7). However, at coefficient of friction so high, the elements’ life cycle cannot be very long. Chances to improve the conditions of operation for the frictional pairs through lubrication are rather limited in the discussed transmissions due to design and kinematic reasons. To ensure the required circumferential frictional force, it is essential to increase the clamping force. Large clamping forces will bring about increases in the contact stresses as well as an increase in the area of contacting surfaces and, as a result, an increase in the drilling friction. Drilling friction puts a limit on efficiency and thus on the transmitted power as a result of significant thermal loads being applied to the small contact
surface areas. Large clamping forces will also cause an increase in loads applied to the transmission bearings, which leads to another significant design problem.

These disadvantages frequently discourage potential users from applying the above-described transmissions in a practical manner. From the reliability and durability points of view, it would be desirable for the frictional elements to operate under conditions of fluid friction where the tangential and circumferential forces are transmitted through an oil film.

As a result of design research, aimed at an increase in the life cycle of the frictional coupling elements, multi-disk frictional transmissions have been developed in which the shortcomings mentioned were significantly reduced. These transmissions can be included in group 2. The element providing frictional coupling is, in this case, a set of conical disks.

Multi-disk frictional transmissions can be divided into two groups:

1. Transmissions in which the active surfaces of both active and passive disks have a positive radius of curvature (disk cone apexes are oriented in opposite directions).

2. Transmissions in which the active surfaces of both active and passive disks have different curvature radii signs (disk cone apexes are oriented in the same direction).

The first type are called multi-disk frictional transmissions with external contact as shown in Fig. 2 (a), and the second type are called multi-disk frictional transmissions with internal contact as shown in Fig. 2 (b) and (c).

Also, multi-disk frictional transmissions, according to the initial contact of the frictional coupling elements, can be further classified as (a) initial line contact type and (b) initial point contact type, as shown in Fig. 3.

All the above-mentioned types of multi-disk stepless transmissions include a packet of disk elements that provide frictional coupling. Multi-disk elements permit multiple frictional contacts between disks resulting in a reduction of the unit frictional force for each pair of disks and a decrease in the axial clamping force. A small axial clamping force allows for a small contact area of the contacting disks. This, of course, decreases
the geometric slip and increases the geometric efficiency of the transmission. Also, through the use of multiple frictional contacts, the energy losses due to geometric slip of the contact surfaces can be significantly reduced compared to other frictional transmission designs.

Fig. 2 Types of multi-disk frictional transmissions: (a) with external frictional coupling; (b) and (c) with internal frictional coupling

Fig. 3 Geometry factors of frictional coupling elements having: (a) initial line contact, (b) initial point contact

However, to select and develop the most rational solution to the frictional transmission problem, a detailed analysis of the phenomena occurring in the contact zone of the frictional coupling elements is necessary.
An analytical and experimental study of these phenomena should evaluate the effects of the particular design parameters on the optimal conditions of operation.

From the known studies related to the analysis of the frictional coupling mechanisms in frictional transmissions, works by O. Lutz [1] should be mentioned. All of the studies done by Lutz have contributed to the knowledge of circular contact area cases. Lutz’s theory, however, cannot be applied to the case when the contact area between rolling elements is not a circular one as, for example, in multi-disk frictional transmissions. In most multi-disk frictional transmissions, the contact areas, in the initial line contact and the initial point contact cases, are not rectangular, elliptical or circular. Rather, they are similar to deformed rectangles or ellipses. An extended analysis of the frictional coupling mechanism in multi-disk stepless transmissions with internal contact where the shapes of the contact areas between the disks’ surfaces are “deformed” rectangles and “deformed” ellipses is presented in reference [2]. The content of this paper is considered to be a significant contribution to the analysis published in reference [4] and, in general, to the field of rolling variators.

2 BASIC KINEMATIC RELATIONS

Kinematic relationships in rolling variators result from the geometric features of the frictional coupling elements. In the multi-disk frictional transmission with initial point contact as shown in Fig. 4, frictional coupling elements are outer and inner disks. The outer disk has rectilinear generating line of the active surface, which makes an angle $\gamma$ with the disk’s axis of rotation. The active surface of the inner (smaller) disk is curvilinear of dual curvature. However, while analyzing kinematic relationships it is reasonable to consider the rolling elements of dual curvature as rolling cones created by median lines of the “contact ring”. Hence, in a general case, the kinematics of the frictional coupling elements is well reflected by the rolling of two cones, as shown in Fig. 5.

Naturally, the transmission of motion without slippage over the length of the contact area is possible only when the apexes of cones $O_1$ and $O_2$ coincide. Otherwise, a geometric slip occurs due to different
circumferential velocities along the line of contact. Rolling without slippage occurs only at point $C$, where the circumferential velocities are equal in both cones. This point is called the rolling point. In the unloaded condition, the rolling point lies close to the center of the line of contact $O$, and the transmission ratio $i_a$ is defined by the rolling radii $R_{01}$ and $R_{02}$.

Application of an external moment to the passive element causes the rolling point $C$ to shift by distance $l$ toward the center of rotation $O_1$ of the active cone. As a result, the rolling radii $R_{01}$ and $R_{02}$ change and the transmission ratio changes as well. A slip occurs, which is called a relative slip. When the

\[ R_{01} = r + \rho \sin \alpha \]
\[ R_{02} = r + \rho \sin \alpha + e \]

Fig. 4 Geometry of frictional coupling elements with initial point contact.
rolling point C shifts significantly beyond the area of contact and the frictional coupling margin, $\kappa$, i.e. ratio of the clamping force to the circumferential frictional force reaches the value of one, a permanent slip occurs [4].

Fig. 5 Kinematic relationships for a frictional pair: (a) in unloaded condition (relative slip $s = 0$); (b) in loaded condition (relative slip $s > 0$); (c) angular velocities
2.1 Transmission Ratio in Unloaded Condition

When unloaded, the contact surfaces do not transmit the desired tangential forces. At the symmetry point of these surfaces, pure rolling occurs (Fig. 5 (a)). The transmission ratio is then defined as

\[ i_0 = \frac{\omega_{01}}{\omega_{02}} = \frac{R_{01}}{R_{02}} \]  

(1)

where \( R_{01} \) and \( R_{02} \) are the distances from the axes of rotation of active and passive elements, respectively, to the center of the contact surface, and \( \omega_{01} \) and \( \omega_{02} \) are angular velocities of active and passive elements, respectively. Introducing the relations \( R_{01} = R_1 \cos \alpha \) and \( R_{02} = R_{II} \cos \alpha \), the transmission ratio in the unloaded condition is

\[ i_0 = \frac{R_{II}}{R_I} \]  

(2)

where \( \alpha \) is the angle between the cone’s base and its generating line, and \( R_I \) and \( R_{II} \) are the lengths of the active and passive cones’ generating lines, respectively.

2.2 Transmission Ratio in Loaded Condition

If a circumferential force is transmitted through the surfaces of contact, the transmission ratio cannot be determined on the basis of the rolling radii \( R_{01} \) and \( R_{02} \). Displacement of the rolling point relative to the center of the contact area causes a change in the angular velocity of the passive element and produces a transmission ratio that depends on the resistive moment and the clamping force. Then, the actual transmission ratio \( i \) is equal to the ratio of the active rolling radii \( R_1 \) and \( R_2 \) (Fig. 5 (b)), which determine the distances between the rolling point C and the axes of rotation of passive and active elements, respectively, so that

\[ i = \frac{\omega_1}{\omega_2} = \frac{R_2}{R_1} \]  

(3)

where \( R_1 = (R_I - l) \cos \alpha \) and \( R_2 = (R_{II} - l) \cos \alpha \). 

11
and thus
\[ i = \frac{R_{\|} - l}{R_i - l} \] (4)

2.3 Relative Slip

The coefficient of relative slip that determines the design features of frictional transmissions is found from
\[ S = 1 - \frac{i}{i} \] (5)

From the relations (2) and (4), we have
\[ S = \left( \frac{R_{\|}}{R_i} - 1 \right) \frac{l}{R_{\|} - l} \] (6)

Introducing a design parameter, \( a \), from Fig. 4, i.e. the length of the contact area along the cones’ generating lines,
\[ S = \left( \frac{R_{\|}}{R_i} - 1 \right) \frac{l}{R_{\|} - l} \] (7)

2.4 Drilling Velocity

The contact surfaces of the rolling elements perform a relative motion of rotation with the angular velocity \( \omega_w \) about the rolling point \( C \) due to the different angular velocities of the cones’ generating lines. This motion can be called “a drilling-type motion” as it is very similar to the motion performed by the tip of a drill relative to the surface being drilled, and the angular velocity \( \omega_w \) can be called the “drilling velocity”.

Projecting the angular velocities \( \omega_1 \) and \( \omega_2 \) onto vectors normal to the generating lines of the cones, we obtain the drilling velocity as the difference of these vectors (see Fig. 5(c):
\[ \omega_w = (\omega_1 - \omega_2) \cos \alpha \] (8)
3.1 Tangential Forces Generated on the Contacting Surfaces of Rolling Elements with Initial Point Contact.

As shown in Fig. 4, if the inner disk has a double curvature, a small contact area occurs near the rolling point. As the contact pressure increases, this area enlarges to an elliptical shape with a curved major axis as shown in Fig. 5. Analytically, the boundary curve of the contact area can be represented by the following equation

\[ \frac{R_I^2 \arctan \left( \frac{x}{y+R_I} \right)}{b^2} + \frac{\left( -R_I + \sqrt{(y+R_I)^2 + x^2} \right)^2}{a^2} = 1 \]  

(9)

for

\[ 0 < \frac{b}{R_I} < \frac{\pi}{2} \quad \text{and} \quad 0 < a < R_I \]

To show that the curve given by equation (9) may be obtained by “bending” an ellipse represented by equation

\[ \frac{u^2}{b^2} + \frac{v^2}{a^2} = 1 \]  

(10)

consider the following transformations

\[ x = (v + R_I) \sin \frac{u}{R_I} ; \quad y = -R_I + (v + R_I) \cos \frac{u}{R_I} \]  

(11)

Transformation (11) maps a closed region, D*, in the Ouv plane onto a closed region, D, in the Oxv plane. It is noted that the Jacobian of this transformation

\[ |J| = \frac{D(x, y)}{D(u, v)} = \frac{v + R_I}{R_I} \]  

(12)
is not equal zero if \( v \neq -R_f \). It is also seen that this transformation maps the segments parallel to the \( v \)-axis of the \( Ouv \) plane into a family of straight line segments passing through the point \( x = 0 \) and \( y = -R_t \) in the \( Oxv \) plane, and the segments parallel to the \( Ou \)-axis into circular arcs centering at \( x = 0 \) and \( y = -R_t \). If \( 0 < b/R_t < \pi/2 \) and \( 0 < a < R_t \), and if the transformed point lies in the area bounded by the ellipse given by (Eq.10), then the Jacobian given by (Eq.12) is positive. Thus, it follows that the transformation (Eq.11) which maps the region bounded by the ellipse (Eq.10) onto the region bounded by the curve (Eq.9) is one-to-one. In other words, the curve (Eq.9) may be obtained by bending the ellipse (Eq.10) with the lengths of both principal axes being held constant. Once the contact area bounded by the curve (Eq.9) is determined, the expression for the tangential force can be obtained in a similar manner as in the linear contact case [4].

![Diagram of contact area](image)

**Fig. 6 Area of contact of a frictional pair with initial point contact.**
Based on Fig. 6, the following relationship can be obtained

\[
\cos \delta = \frac{R_l + y - \Delta}{\sqrt{(R_l + y - \Delta)^2 + x^2}}
\]  

(13)

The contact pressure at any point inside the contact area bounded by the curve given by the equation (9) is expressed as

\[
p(x, y) = p_0 \sqrt{1 - \frac{R_l^2 \arctan^2 \left( \frac{x}{R_l + y} \right)}{b^2} - \left[ -R_l + \sqrt{(R_l + y)^2 + x^2} \right]^2}
\]

(14)

where \( p_0 = \frac{3}{2 \pi ab} \) and the effective tangential force, \( T_u \), is given by

\[
T_u = \mu p_0 \int_{D} p(x, y) \cos \delta \, dx \, dy
\]

(15)

where \( D \) represents the region bounded by the curve given by equation (9) over which the integration is performed.

![Mechanism of generation of tangential forces on the surface of a frictional pair.](image)

**Fig. 7** Mechanism of generation of tangential forces on the surface of a frictional pair.
Substituting the values of \( \cos \delta \) and \( p(x, y) \) from equation (13) and (14), respectively, into equation (15) yields

\[
T_U = \mu p_0 \int_D \sqrt{1 - \frac{R_i^2 \arctan^2\left(\frac{x}{R_i} + y\right)}{b^2}} \frac{\left[-R_i + \sqrt{(R_i + y)^2 + x^2}\right]^2}{a^2} \cdot \frac{R_i + y - \Delta}{\sqrt{(R_i + y - \Delta)^2 + x^2}} \, dy \, dx
\]  

(16)

Using equation (11) and the Jacobian of the transformation given by equation (12), equation (16) becomes

\[
T_U = \mu p_0 \int_D \sqrt{1 - \frac{u^2 - v^2}{a^2}} \cdot \frac{(v + R_i) \cos u - \Delta}{R_i} \cdot \frac{v + R_i}{R_i} \, dudv
\]  

(17)

where \( D^* \) denotes the region bounded by the ellipse given by equation (10).

Introducing the variables

\[
u = b \xi; \quad \nu = a \eta
\]  

(18)

and noting that the Jacobian of the transformation

\[
J = \frac{D(u, v)}{D(\xi, \eta)} = ab
\]

it is easily verified that equation (18) maps an ellipse \( \frac{u^2}{b^2} + \frac{v^2}{a^2} = 1 \) into a circle, \( \xi^2 + \eta^2 = 1 \).

Using equation (18), equation (17) becomes

\[
T_U = \mu p_0 ab \int_K \sqrt{1 - \xi^2 - \eta^2} \cdot \frac{(a \eta + R_i) \cos \frac{b}{R_i} \xi - \Delta}{\sqrt{(a \eta + R_i)^2 - 2\Delta(a \eta + R_i) \cos \frac{b}{R_i} \xi + \Delta^2}} \cdot \frac{a \eta + R_i}{R_i} \, d\xi \, d\eta
\]  

(19)

where \( K \) denotes the region bounded by the circle, \( \xi^2 + \eta^2 = 1 \). The next transformation helps express the integrand in terms of the given parameters \( \frac{b}{a}, \frac{R_i}{a}, \frac{l}{a} \) and includes the observation that it is an even
function of $\xi$. $T_u = 2\mu P_a^2 \int_0^{\frac{R_1}{a}} \int_{1-\xi^2}^{\frac{R_1}{a}} \sqrt{1-\xi^2-\eta^2} d\xi d\eta$

\[
\left( \eta + \frac{R_1}{a} \right) \cos \frac{h}{R_1} \xi - \left( \frac{R_1}{a} - \frac{l}{a} \right) \left( \eta + \frac{R_1}{a} \right) \right] d\eta
\]

\[
\sqrt{\left( \eta + \frac{R_1}{a} \right)^2 - 2\left( \eta + \frac{R_1}{a} \right) \left( \eta + \frac{R_1}{a} \right) \cos \frac{h}{R_1} \xi + \left( \frac{R_1}{a} - \frac{l}{a} \right)^2}
\]

Substituting $\frac{R_1}{a} = m$; $\frac{l}{a} = n$; $\frac{h}{a} = w$, the final expression for the effective tangential force for the initial point contact case is obtained from equation (19):

\[
T_u = 2\mu P_a^2 \int_0^m \int_{1-\xi^2}^{\frac{m}{a}} \sqrt{1-\xi^2-\eta^2} d\xi d\eta
\]

\[
\left( \eta + m \right)^2 \cos \frac{w}{m} \xi - \left( m - n \right) \left( \eta + m \right) \right] d\eta
\]

\[
\sqrt{\left( \eta + m \right)^2 - 2\left( \eta + m \right) \left( m - n \right) \cos \frac{w}{m} \xi + \left( m - n \right)^2}
\]

3.2 Tangential Force Utilization Coefficient

One of the basic requirements for proper operation of a frictional transmission is the minimization of the relative slip of the friction pair. The slip depends primarily upon the value of the frictional coupling margin coefficient, $K$ [4]. This parameter is defined as the ratio of the force, $T_u$, to the clamping force, $P$, namely,

\[
k = \frac{T_u}{P}
\]

In order to eliminate an extensive slip in frictional transmissions, the expected frictional margin, $K$, should be smaller than the actual coefficient of friction, $\mu$, i.e.,

\[
\frac{K}{\mu} < 1
\]

This ratio is called here the tangential force utilization coefficient [4]

\[
U_{\text{fr}} = \frac{K}{\mu}
\]

Using the definition of $K$ given by Eq. (22), it follows that
Substituting for \( T_u \) from Eq. (21) and for \( p_0 = \frac{3}{2} \frac{P}{\pi ab} \), we can transform Eq. (24) to get

\[
U_{TP} = \frac{T_u}{p_\mu}
\]  

(24)

Substituting for \( T_u \) from Eq. (21) and for \( p_0 = \frac{3}{2} \frac{P}{\pi ab} \), we can transform Eq. (24) to get

\[
U_{TP} = \frac{3}{\pi m} \int_0^1 \int_{\sqrt{1-\xi^2}}^{\sqrt{1-\eta^2}} \frac{(\eta + m) \cos \frac{w}{m} \xi - (m-n)(\eta + m)}{(\eta + m)^2 - 2(\eta + m)(m-n)\cos \frac{w}{m} \xi + (m-n)^2} \, d\eta
\]  

(25)

3.3 Power Loss Coefficient

The friction moment on the contacting surfaces which is due to the drilling-type motion of these surfaces about the rolling point \( C \) (see Fig. 7) can be defined as

\[
dM_{DP} = dT_{\xi \cdot} \rho_C
\]  

(26)

This moment is called the drilling-type friction moment. Referring to Fig. 7, the drilling friction moment for the initial point contact case can be expressed as

\[
dM_{DP} = \mu \int_D p(x, y) \cos \delta \rho_c \, dx \, dy
\]  

(27)

where

\[
\rho_c = \sqrt{R_i + y - \Delta} + x^2
\]

Following the same derivations of the tangential force presented in Section 4.1, yields

\[
M_{DP} = 2\mu p_0 a^2 \frac{w}{m} \int_0^1 \int_{\sqrt{1-\xi^2}}^{\sqrt{1-\eta^2}} \frac{1}{\sqrt{1-\xi^2 - \eta^2}}
\]

\[
\times \sqrt{(\eta + m)^2 - 2(\eta + m)(m-n)\cos \frac{w}{m} \xi + (m-n)^2} \, d\eta
\]  

(28)

Substituting for the maximum contact pressure

\[
p_0 = \frac{3}{2} \frac{P}{\pi ab}
\]

into equation (28) and dividing both sides by \( P \mu a \), the coefficient of the power loss for the initial point contact case is obtained as
Moment Arm of the Resultant Tangential Force

Referring to Fig. 8 and following similar derivations as in paper [4], the moment arm of the resultant tangential force for the initial point contact case is given as

\[ l_{NP} = a \frac{U_{DP}}{U_{TP}} \]  

(30)

Introducing the parameter \( \frac{b}{a} \), gives

\[ \frac{\sqrt{ab}}{l_{NP}} = b \frac{U_{TP}}{a U_{DP}} \]  

(31)

Fig. 8 Moment arm of the resultant frictional force
3.5 Geometric Efficiency of the Frictional Pair

Following the same derivations for the efficiency of the frictional pair with initial line contact, as presented in paper [4], the efficiency of the frictional pair with initial point contact is given as

\[
\eta = \frac{1}{\sqrt{\frac{a}{b} \left( \frac{R_i}{a} - \frac{l}{a} \right)}} + \frac{\sqrt{ab}}{l_{NP}}
\]

Based on Eq. 32, it can be observed that the efficiency of the frictional pair, which depends on the amount of power loss due to drilling-type friction (see Eq. 31), is also conditioned to a large extent by the size of the contact surface determined by the parameters \(a\) and \(b\).

4 SUMMARY AND CONCLUSIONS

4.1 Summary of Numerical Results

The relationships determining the tangential force utilization coefficient \(U_{TP}\) (Eq. 25) and the power loss coefficient \(U_{DP}\) (Eq. 29) are expressed in the form of elliptical integrals and cannot be evaluated in closed form. Therefore, by introducing into the expressions under the integrals the parameters of the contacting surfaces in rational form as \(w = b/a\), \(n = l/a\), and \(m = R_i/a\) and assigning specific numerical values to these parameters, it allows one to perform numerical integration. The results obtained are presented in graphical form as shown in Figs. 9, 10, and 11.
Fig. 9 shows the variations of the values of the parameter \( U_{TP} \) vs. the location of the rolling point \((l/a)\) and the ratio of the dimensions of the contact surface \((b/a)\) for \( m = R_i/a = 100 \). As this figure indicates, the values of the tangential force utilization coefficient \( U_{TP} \) are increasing with the increase in the value of the ratio \( l/a \). At the given value of \( l/a \), the coefficient \( U_{TP} \) remains almost unchanged in spite of the variation in the values of \( b/a \) from 0.1 to 1.0. Based on this observation, it can be concluded that if the width of the contact surface, \( a \), is greater than the length, \( b \), that is, if \( b/a \leq 1 \), the projections of the differential tangential force \((dT_{12})\) on the \( O-x \) axis (see Fig. 7) are significantly greater than the projections of this force on the \( O-y \) axis. If the width of the contact surface, \( a \), is much greater than its length, \( b \), (five or more times), the projections of the differential tangential force \((dT_{12})\) on the \( O-y \) axis are equal to zero under the
assumption that the contact pressures are uniformly distributed along the average line of contact. With the increase of the ratio \( b/a \), the coefficient \( U_{TP} \) reaches its maximum value (that is, \( U_{TP} \) approaches unity) when the ratio \( l/a \) increases significantly.

![Graph showing variation of power loss coefficient \( U_{DP} \) versus ratios \( l/a \) and \( b/a \) for \( R_l/a = 100 \).](image)

**Fig. 10 Variation of the values of the power loss coefficient \( (U_{DP}) \) versus the ratios \( l/a \) and \( b/a \) for \( R_l/a = 100 \).**

Fig. 10 shows the variation in the values of the power loss coefficient \( U_{DP} \). As this figure indicates, the value of the coefficient \( U_{DP} \) (and at the same time the percentage of the power loss due to drilling friction) depends mainly on the size of the contact surface, that is, on the ratio \( b/a \). Thus, in the design of rolling variators, in order to decrease the power loss due to drilling-type friction, it is necessary to keep the ratio \( b/a \) small. The influence of the parameter \( m = R_l/a \) on the variations in the values of the coefficients \( U_{TP} \)
and $U_{DP}$ is rather small.

Fig. 11 Variation of the values of the parameter $\frac{\sqrt{ab}}{l_{NP}}$ versus the ratios $l/a$ and $b/a$ for $R_i/a = 100$.

Fig. 11 illustrates the variation in the values of the moment arm of the resultant tangential force, that is, in the values of the parameter $\frac{\sqrt{ab}}{l_{NP}}$ (see Eq. 31). The values of this parameter allow one to evaluate the efficiency of the frictional couple using Eq. 32. The value of this parameter depends on the values of the coefficients $U_{TP}$ and $U_{DP}$. 
Fig. 12  Geometric efficiency $\eta$ of the frictional pair as a function of the ratios $l/a$ and $b/a$

for $R_I/a = 60$, and $R_{II}/a = 120$.

Figure 12 shows the efficiency of the frictional pair as a function of the geometrical factors $l/a$, $b/a$, $R_I/a$, and $R_{II}/a$. The efficiency of the frictional pair depends mainly on the values of the tangential force utilization coefficient, $U_{TP}$, the power loss coefficient, $U_{DP}$, and on the value of the parameter $\frac{\sqrt{ab}}{l_{NP}}$, which represents the moment arm of the effective tangential force $T_U$ (see Eq. 21).
4.2 Conclusions

An analytical study has been performed to investigate the frictional coupling mechanism in multi-disc stepless transmissions with initial point contact. From this study, which provides valuable information for the design of this type of rolling variators, the following conclusions can be drawn.

1. As the result of this study, and analytical relationship for the basic parameters characterizing frictional coupling mechanisms in multi-disc stepless transmissions with initial point contact has been obtained.

2. Based on the numerical values of these parameters, that is, the tangential force utilization coefficient $U_{TP}$, and the power loss coefficient $U_{DP}$, the moment arm of the resultant tangential friction force can be evaluated and this, in turn, allows one to evaluate the efficiency of the friction pair.

3. The efficiency of the friction pair depends on the ratio $b/a$, that is, on the circumferential length, $b$, and the radial width, $a$, of the contact surface as well as on the location of the rolling point, that is, on the ratio $l/a$. The location of the rolling point, $C$, depends on the amount of the transmitted torque.

4. The presented graphical analysis of the numerical values of $U_{TP}$, $U_{DP}$, and $\frac{\sqrt{ab}}{l_{NP}}$ for a wide range of the ratios $b/a$, $l/a$, and $R_i/a$, provides valuable information for the optimal design of rolling variators.
5. The most effective way to decrease the power loss coefficient $U_{DP}$, and thereby, to increase the efficiency of the friction pair, is to decrease the coefficient of the drilling-type friction, $\mu_D$. This can be achieved by selection of proper geometry of the friction elements’ contacting surfaces [3].

6. Proper geometry of the contacting surfaces of the friction elements may provide conditions for hydrodynamic lubrication. This in turn will increase substantially the efficiency of the friction pair by decreasing the power loss coefficient [3].

REFERENCES


NOMENCLATURE

\( a \) = half of the width of the contact area measured in the radial direction,

\( b \) = half of the length of contact area measured in the circumferential direction,

\( C \) = the rolling point,

\( dT_{12} \) = differential tangential force generated at the contact surface,
\( i \) = actual transmission ratio in loaded condition,

\( i_o \) = transmission ratio in unloaded condition,

\( l \) = displacement of the rolling point,

\( l_{NP} \) = arm of the resultant tangential force with respect to the rolling point, \( C \),

\( M_w \) = frictional moment resulting from the drilling-type motion of the contacting surfaces,

\( N_1 \) = the overall power,

\( N_w \) = power loss due to the drilling-type friction,

\( P \) = clamping force,

\( p_o \) = contact pressure,

\( R_{01} \) and \( R_{02} \) = the distances from the axes of rotation of active and passive elements, respectively, to the center of the contact surface,

\( R_1 \) and \( R_2 \) = the distances from the axes of rotation of active and passive elements, respectively, to the location of the rolling point, \( C \),

\( R_1^\prime \) and \( R_2^\prime \) = the distances from the apexes of the disk surface cones, \( \alpha \), and \( \beta \), to the center of the contact surface along the cones’ generating lines,

\( S \) = relative slip,

\( T_u \) = the resultant circumferential tangential force,

\( U_{TP} \) = the tangential force utilization coefficient,

\( U_{PF} \) = power loss coefficient,

\( \alpha \) = angle between the cone’s base and its generating line,

\( \beta \) = half of the angular width of the contact surface,

\( \gamma \) = half of the cone angle of the frictional disks,

\( \kappa \) = frictional coupling margin,

\( \mu \) = coefficient of friction between the contacting surfaces of the frictional disks,

\( \omega_{01} \) and \( \omega_{02} \) = angular velocities of the active and passive elements, respectively,