I received a SAIF Grant for the 2010-2011 academic year titled “Development of a Lens Characterization Instrument”. The key outcomes from this grant are described below.

The goal of this project was to develop a novel instrument for measuring the focal length of a lens. The key attributes of this instrument were that it be automated, have no moving parts, not require any adjustments from the user, not require any imaging (or focusing) either electronically or by a human, suitable for positive and negative lenses over a fairly broad focal length range (25mm < |f| < 1000 mm), include an LCD readout, be compact, simple to use, and accurate. Such an instrument has been designed, prototyped, and achieves all these goals. A complete description of this work is attached.

Along the way, I learned a great deal about optical systems, optical design, optical instrumentation, and optical calculation techniques. In particular, I discovered a great many techniques which have been developed for characterizing lenses (and optical systems in general). Fortunately, this research also convinced me that my ‘idea’ is indeed novel. The benefit to me is immense. As someone who has studied optics for many years, it was fun to pursue something different, research the subject matter, and learn lots of new things. I have written a paper regarding this project and submitted on August 3 to the “Journal of Optics and Lasers in Engineering”. The paper is currently under review. Attached is a copy of that paper. Additionally, my research on this project has sparked a few other ideas which I’m currently pursuing.

At the beginning of this project I had a student work on one aspect of this project for independent study. I know he learned a variety of new things (e.g. using a microcontroller and LCD as well as interfacing a PSD). This Fall, I will have another student for independent study prepare a couple of experiments for our optics lab course. These experiments are based on techniques I learned as part of my research on this project. I will also weave this material into the optics lecture which I teach in the Spring. Last Spring my optics students found that the project I was working on, this one, incorporated techniques which I was presenting to them as part of the optics course. Apparently students are impressed when a professor can actually walk the talk.

In summary, my knowledge of optics expanded, a publication in an international optics journal is forthcoming, two students acquired valuable research experience under my tutelage, and our optics course will benefit from new material.

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A Simple Automated Instrument for Measuring the Focal Length of a Misaligned Lens

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An instrument has been prototyped for the purpose of determining the effective focal length of a spherical lens. The instrument employs a 2D position sensitive detector (PSD), two diode laser modules, a microcontroller, and LCD readout. The system is designed to be automated, immune to modest misalignment of the test lens, require no mechanical adjustments or image processing, accurate for spherical lenses with positive or negative focal lengths in the range 25 mm < |f| < 1000 mm, and lens diameters no less than 25 mm. The user only needs to insert the test lens. The optical design of the system is motivated by simulations which account for lateral, longitudinal, and angular misalignment of the test lens.

Key words: focal length, position sensitive detector

1. Introduction

The primary motivation for this work was to develop a simple, automated instrument able to accurately determine the effective focal length of positive and negative lenses over a relatively broad focal length range (i.e. a range typical of general purpose lenses found in undergraduate
optical laboratories). Furthermore, the instrument should be user-friendly to the extent the user only needs to insert the lens. No adjustments such as alignment or focusing should be necessary nor image processing (which might require sophisticated software and a computer). The instrument should be compact, self-contained, and not require any peripheral hardware.

Measuring the focal length of a lens is certainly an old science. There are the Bessel’s and Abbe’s methods [1], Newton’s method [2], and somewhat more sophisticated methods including the Nodal slide which can determine the Cardinal points and the focimeter which is typically used to measure ophthalmic lenses [3]. While only an optical bench is necessary, all these methods require moving one or more devices (e.g. screen or lens) and determining image focus. Automating any one of these methods would require mechanical actuation and image processing. An example of this is an automated focimeter [4]. Other methods for determining lens focal length include interferometric techniques [5-11], techniques which employ gratings or rulings [12-15], a z-scan approach [16], fiber arrays [17], the Lau effect [18], and Gaussian beam imaging [19]. These methods also require some form of mechanical alignment and image processing (electronic or human perception). In order to quantify lens aberrations, the Hartmann test [3, 20-21] has proven a standard method. Utilizing wavefront sensing technologies [22] there are very sophisticated instruments available which essentially determine “all” the properties of a lens [23-25].

The method discussed here is similar to a ‘simplified’ Hartmann test [26] and ‘scanning’ Hartmann test [27]. However, this approach is unique in that it employs two lasers, a single large area PSD [28,29], has no moving elements, does not require image processing, and is solely for the purpose of determining the effective focal length. Thus while it is limited in terms of making only a single type measurement, its’ simplicity lends itself to low cost, compact size, and
portability. Additionally, this system allows for a modest misalignment of the test lens which enhances the ‘user-friendliness’ of such an instrument. The key principles lie in averaging the output signals from the two lasers which negates lateral decentering of the test lens as well as employing a Fourier lens between the test lens and PSD which negates longitudinal misalignment. These characteristics further distinguish this instrument from other strategies.

The intended users of this instrument would be those who require a relatively inexpensive automated instrument which can quickly and easily measure the effective focal length of positive and negative lenses; if for no other reason than maintaining organization in an optical laboratory which serves many individuals handling many lenses.

2. Design and Simulation

The basic idea is quite simple. Referring to Fig.1, consider a single ray directed towards a misaligned test lens with effective focal length $f_x$ and detected by a PSD centered square with the optic axis. The coordinate system shown is conventional with the positive z-axis along the direction of light travel and when viewing in this direction the y-axis positive up and the x-axis positive to the right [30].

![Fig.1 A simple system utilizing a single laser, test lens, and PSD](image_url)
The transformation of a ray is analyzed using 3x3 ray matrices for a cylindrically symmetric, nonastigmatic system within the paraxial domain [31-33]. For these systems the ray transformation matrices are the same for both lateral coordinates. Also, a lateral misalignment in the x-direction only effects the x-coordinate and a lateral misalignment in the y-direction only effects the y-coordinate. As such, a two-dimensional treatment will suffice. Arbitrarily, for the initial analysis, the y-coordinate will be analyzed. Let the ray position and inclination angle at the input plane along the y-axis be denoted \( \{ y_i, \alpha_i \} \) respectively and similarly at the output plane \( \{ y_o, \alpha_o \} \). Misalignment of the test lens is characterized by a longitudinal translation, \( \delta z \), lateral translation \( \delta y \), and tilt (rotation) about the x-axis, \( \delta \theta_x \). (The misalignment illustrated in Fig.1 would be characterized by \( \delta z > 0, \delta y > 0, \) and \( \delta \theta_x > 0 \).) For the simulations, the test lens is assumed to be thin. Of course, no lens is truly ‘thin’, however, the modeling discussed here is merely used as a guide for design not to exactly predict the performance of the actual system.

The transformation of the ray parameters are described by the following 3x3 matrices,

\[
\begin{pmatrix}
\alpha_o \\
y_o \\
1
\end{pmatrix} =
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & d_2 - \delta \varepsilon & 0 \\
0 & 1 & \delta \theta_x \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 \\
1 & 0 & 0 \\
0 & 1 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0 & -\delta y \\
1 & 0 & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 \\
1 & 0 & 0 \\
0 & 1 & 1
\end{pmatrix}
\begin{pmatrix}
y_i \\
\delta \varepsilon \\
1
\end{pmatrix}
\]

(1)

Carrying out the matrix multiplication shows that the output ray position is,

\[
y_o = y_i \left( 1 - \frac{d_2 - \delta \varepsilon}{f_x} \right) + \alpha_i \left[ d_2 - \delta \varepsilon + (d_1 + \delta \varepsilon)(1 - \frac{d_2 - \delta \varepsilon}{f_x}) + \delta y \left( \frac{d_2 + \delta \varepsilon}{f_x} \right) \right]
\]

(2)

which is independent of tilt (rotation). Next, we consider the longitudinal misalignment specified by the terms involving \( \delta \varepsilon \). Referring to Fig.2, a positive lens with focal length \( f \) is positioned a distance \( d_2 \) from the test lens and the PSD positioned a distance \( d_3 \) equal to \( f \) from this lens, hence referred to as a Fourier lens. (Also shown in Fig.2 is a telescope used to reduce the separation between the two laser beams in order that the beam spots at the PSD fall within the
PSD aperture. This is discussed further in the section 4.) Ignoring the telescope for the moment, to include the lens $f$ (assumed ‘thin’) and the translation $d_3$, the following matrices are added to the left of the matrices in (1),

$$
\begin{pmatrix}
1 & d_3 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 \\
-1/f & 1 & 0 \\
0 & 0 & 1 \\
\end{pmatrix}
$$

(3)

Upon setting $d_3 = f$ and resolving the product of matrices, the output position becomes,

$$
y_o = -\frac{f}{f_x} [y_i - \delta y - \alpha_i (1 - d_t - \delta \alpha)]
$$

(4)

which while simpler than (2), but at the expense of adding a lens, has not eliminated the effects of misalignment. However, consider now using two parallel rays with input ray parameters $y_{i1} = -y_{i2} = y_i$ and $\alpha_{i1} = \alpha_{i2} = \alpha_i$ (i.e. two parallel rays symmetric about the y-axis). The average of the output ray positions, $y_{o1}$ and $-y_{o2}$, henceforth designated $Y$, is given by the simple result,

$$
Y = \frac{1}{2} (y_{o1} - y_{o2}) = -y_i \frac{f}{f_x}
$$

(5)

---

Fig.2 The final optical system showing the addition of a second laser, telescope, and Fourier lens.
Albeit not shown, for $d_3 \neq f$ and/or $\alpha_{i1} \neq \alpha_{i2}$ the result is significantly more complicated.

The final result expressed by (5) satisfies the goal of an output position independent of misalignment of the test lens including lateral and longitudinal translation as well as tilt (as long as the two input rays are parallel and enter the optical system symmetric about the optic axis). Of course, all of the preceding is only valid within the paraxial limit and for ‘thin’ lenses.

3. Simulation Results and Discussion

As an example, consider the following scenario: $y_{i1} = -y_{i2} = 1.5$ mm, $\alpha_{i1} = \alpha_{i2} = 0$, $d_2 = 65$ mm, and $d_3 = f = 50$ mm. (The rationale for the values chosen are discussed in section 4 as is the role of the telescope. Also, the output of the telescope is now the input plane of the system.) Fig.3 shows the output coordinates $y_{o1}, y_{o2}, -y_{o2}$, and $Y$ as a function of $1/f_x$ for $\delta y$ equal to 1.0 mm and 2.0 mm. The most important result which cannot be gleaned from this graph is that for no misalignment of the test lens, the output coordinates $y_{o1}$ and $-y_{o2}$ exactly match the quantity $Y$. (In other words, $y_{o1}, -y_{o2}$, and $Y$ are the same.) This is the reason for formulating the quantity $Y$.

Fig.3  Theoretical simulation of the output position for each of the two laser beams, $y_{o1}$ and $y_{o2}$, the negative of $y_{o2}$, and the average quantity $Y = (y_{o1} - y_{o2})/2$. The lateral misalignment is a) $\delta y = 1$ mm and b) $\delta y = 2$ mm. For no misalignment, the output positions are identical to $Y$. 
Clearly a misalignment of the lens transverse to the optic axis has a significant affect, however this misalignment may be negated by employing two rays as described. The same analysis could be made for the ray coordinates along the x-axis and misalignment along the x-axis. The result is that a similar quantity $X$ is given as,

$$X = - x_i \frac{f}{f_s}$$

(6)

Summarizing, to accommodate an arbitrary lateral misalignment in the xy-plane, the two rays need only have ‘symmetric’ input ray coordinates, $\{x_{i1}, y_{i1}\} = \{-x_{i2}, -y_{i2}\}$ and be parallel, $\alpha_{i1} = \alpha_{i2}$.

4. Experimental Apparatus

The lasers are from Thorlabs (#CPS182). These lasers have an elliptical spot, beam divergence < 2 mrad, and 11 mm diameter housing. The wavelength is 635 nm. The laser housing is electrically conducting and at V+ thus when two are mounted side-by-side, they must be insulated from each other (so that they can be turned on/off separately). Experimentally, the two beam centers were separated by 17 mm and a nonconducting mount was fabricated in order to insulate them from one another as well as render their beams parallel. In order to satisfy the PSD aperture, it was necessary to implement a telescope to reduce the laser beam separation. The telescope consist of a +100 mm lens followed by a − 25 mm lens separated by 75 mm which results in a magnification $-1/4$. This renders the two beam centers separated by $17/4$ mm = 4.25 mm. The two lasers are directed symmetrically towards the diagonal of the PSD parallel to the optic axis so that the input coordinates are $\{x_{i1}, y_{i1}\} = \{-x_{i2}, -y_{i2}\} = \{1.5$ mm, 1.5 mm$\}$ and $\alpha_{i1} = \alpha_{i2} = 0$. 
The detector is a Hamamatsu (#S1200) two-dimensional PSD with an active area 13 mm x 13 mm. The singlet test lenses are from Thorlabs (#ESK-53A). All positive singlets are biconvex, the three shortest negative focal length lenses biconcave, and the other negative lenses meniscus. The test lens is mounted on a $xz\theta_y$ stage and because of the stage’s physical size, $d_2$ was set to 65 mm. Any closer was mechanically inconvenient and any further the aperture of the system would prove too small (specifically for the shorter negative focal length lenses). Note: the simulations focused on the y-coordinate and a lateral misalignment in the y-direction. However, experimentally a lateral misalignment in the x-direction will be used. Because the effects in the two directions are independent there is no inconsistency between the simulations and experiment. It is simply easier to experimentally translate the test lens in the horizontal direction than in the vertical direction. Of course the simulations could have used the x-coordinate, but then the illustrations given by Fig.1-2 would require a 3D rendering (unless the x-coordinate is defined positive up and the y-coordinate defined positive to the left, but that would be unconventional).

In order to automate the instrument, a microcontroller and 2-line LCD are implemented [33]. The PSD output signals are conditioned using a single stage current-to-voltage converter, the microcontroller processes these signals to produce the average values $X$ and $Y$, and then uses a heuristic (or phenomenological) model to predict the value $f_x$. This value is then written to the LCD. The microcontroller also controls the two lasers; switching them on/off one at a time for the two independent measurements.

The test lenses were inserted using a self-centering lens holder. The jaws of this holder are notched. For all lenses tested, the physical center of the lens was position at these notches. Of
course the physical center does not necessarily have the same relation to the principle planes for all lenses, but this method was followed for consistency nonetheless.

5. Experimental Results and Discussion

Anticipating from (5) and (6) that the signals $X$ and $Y$ vary as $1/f_x$, the following heuristic model was used,

$$X = a_x + \frac{b_x}{f_x} \text{ and } Y = a_y + \frac{b_y}{f_x}$$

(7)

The fitting parameters $a$ and $b$ were determined using the method of least-squares. The parameter $a$ accounts for a small decentering of the two laser beams relative to the PSD. Ideally the same fitting parameters would suffice for both positive and negative values of $f_x$ as well as for $X$ and $Y$, however, a better result was obtained if fitting parameters were determined separately for positive and negative lenses as well as $X$ and $Y$. Thus there are four sets of fitting parameters; a pair for $f_x > 0$ and a different pair for $f_x < 0$. The two predictions for a positive lens are averaged and the two predictions for a negative lens are averaged. The parameters are shown in Table I.

<table>
<thead>
<tr>
<th></th>
<th>a (mm)</th>
<th>b (mm^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X (f_x &gt; 0)$</td>
<td>0.134</td>
<td>-95.7</td>
</tr>
<tr>
<td>$X (f_x &lt; 0)$</td>
<td>0.136</td>
<td>-69.4</td>
</tr>
<tr>
<td>$Y (f_x &gt; 0)$</td>
<td>-0.0210</td>
<td>-86.3</td>
</tr>
<tr>
<td>$Y (f_x &lt; 0)$</td>
<td>-0.0482</td>
<td>-70.0</td>
</tr>
</tbody>
</table>

Table I    Fitting parameters used in heuristic model
Fig. 4  a) Measured values of a) $X$ and b) $Y$ (shown as •), as well as the heuristic model (solid line) for different test lenses with focal length, $f_x$ (as claimed by manufacturer).

Fig. 5  Measured value of focal length, $f_{\text{meas}}$ (shown as •), for the various test lenses with focal length, $f_x = f_{\text{mfr}}$. For ease of comparison a 45° line is shown (solid line).
Fig. 6  Measured value of focal length, $f_{\text{meas}}$ (shown as •), for the various test lenses with focal length, $f_x = f_{\text{mfr}}$. For ease of comparison a 45° line is shown (solid line). Figures a-c are for the positive lenses and figures d-f the negative lenses. All plots use a log-log scale and the type of misalignment and average percent difference are indicated. Each graph has two values of misalignment in the given direction distinguished by the symbols • and □.

Fig. 5 shows the result upon programming the microcontroller with the heuristic model and fitting parameters and testing the system. The manufacturer’s value, $f_{\text{mfr}}$, is the effective focal length quoted by the manufacturer, claimed to be accurate to within 1% and determined at
the wavelength 532 nm. The value measured by the instrument, \( f_{\text{meas}} \), utilizes the heuristic model and is given by the average values using both the \( X \) and \( Y \) quantities,

\[
f_{\text{meas}} = \frac{1}{2} \left( \frac{b_x}{X - a_x} + \frac{b_y}{Y - a_y} \right)
\]  

This is the value displayed on the LCD. The percent difference between \( f_{\text{mfr}} \) and \( f_{\text{meas}} \), defined as

\[
\left| \frac{2 \times (f_{\text{mfr}} - f_{\text{meas}})}{(f_{\text{mfr}} + f_{\text{meas}})} \right| \times 100\%,
\]

averaged over all test lenses is 0.99%.

Fig.6a-c shows the effect of misalignment for the positive test lenses and Fig.6d-f shows the effect of the same misalignment for the negative test lenses. The plots use a log-log scale in order to separate out the data points. Because this cannot be done for negative values, the results for the positive lenses are shown separately from the results for the negative lenses and for the negative lenses, \( -f_{\text{meas}} \), is plotted against \( -f_{\text{mfr}} \). Fig.6a and Fig.6d show the results for a lateral misalignment, Fig.6b and Fig.6e a tilt misalignment, and Fig.6c and Fig.6f a longitudinal misalignment. The average percent difference between \( f_{\text{meas}} \) and \( f_{\text{mfr}} \) are indicated. With the exception of lateral misalignment for the shorter negative lenses, the difference varies between 1 – 10%. Fig.6d makes clear for a negative focal length lens with a focal length ‘shorter’ than -100 mm, a lateral misalignment as small as 1 mm is significant. Whether or not the difference is acceptable as well as whether or not the degree of misalignment is realistic will depend on the application and means by which the lens is inserted in to the instrument. If the user can accept a 10% difference, a simple CD/DVD-like tray mechanism might serve well for inserting the lens. If greater accuracy is needed, a self-centering lens holder should be used. For the shorter negative lenses, the lens should be aligned.

6. Ancillary Comments

The choice of components was based on availability and affordability. A small budget allowed for the purchase of an inexpensive microcontroller and LCD, however the lasers, PSD,
and test lenses were chosen simply based on availability. A few changes would very likely improve the end result. For example, using lasers with round beam spots as opposed to elliptical would improve the symmetry of the spot at the PSD. The response of this type of PSD is nonlinear over the active area and thus the more asymmetric the beam spot, the less linear the response. Also, the telescope could be eliminated if the two laser beams could be made to be closer together. One set of smaller laser modules were tested, however their beams were not collinear with their housings making parallelism between the two beams very difficult. Also, the telescope used here to minify the beam separation simultaneously magnifies the divergence of each beam by the same factor. This challenges the aperture of the system.

The choice of lenses suited well the intent of this instrument which was to be compatible with lens types typically found in an undergraduate optics inventory. While the results were not shown here, larger lens diameters were tested with excellent results (e.g. 1.5” and 2” diameters). As for the lens mount used, a self-centering lens holder was used. This holder was effective at minimizing a lateral decentering, however because the location of the principle planes is not known a longitudinal uncertainty on the order ±1 mm is unavoidable. Fig.6c and Fig.6f show the average difference for a longitudinal translation ± 5 mm and while not shown the difference is negligible for a translation on the order ±1 mm. The original idea for a lens holder was to implement a kind of tray similar to a CD/DVD tray. This would be a user-friendly way of inserting a test lens. This was one motivation for considering a design which would be immune to a modest misalignment of the test lens.

Lastly, it is conceivable that this system could be made relatively compact and possibly portable. There are few components and the electrical needs minimal and of very low power.
This is essentially because there are no moving parts or image processing needs (which might otherwise require sophisticated software and a computer).

7. Conclusion

A novel instrument has been prototyped capable of determining to within an average percent difference < 1% the focal length of positive and negative lenses in the range 25 mm < |f| < 1000 mm. If the test lens is misaligned in any direction including tilt the ‘error’ of course grows, however the extent of the error may be deemed small depending on the nature of misalignment and desired accuracy. The instrument is generally tolerant of modest misalignment of the test lens suitable for identification of general purposes lenses. The prototype is automated, has no alignment or focusing requirements, is a stand-alone design, and lends itself to a compact and portable instrument.

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References


[12] Daniel Malacara-Doblado, Didia P. Salas-Peimbert, and Gerardo Trujillo-Schiaffino, 
Measuring the effective focal length and the wavefront aberrations of a lens system, *Optical 
Engineering* 49(5), 2010.


[14] Rajpal S. Sirohi, Harish Kumar, and Narinder K. Jain, Focal length measurement using 

[15] Xiangqun Cao, Song Shen, and Jiwu Chen, Focal length measurements with a three-grating 

[16] Yasser Abdelaziez and Partha P. Banerjee, A simple focal-length measurement technique for 

[17] Pei Huang, High-precision measurement of effective focal length with single-mode fiber 

[18] S. Prakash, S. Singh, and A. Verma, A low cost technique for automated measurement of 
2033-2042.

[19] Alma A. Camacho P., Cristina E. Solano, Geminiano Martinez-Ponce, and Rosario Baltazar, 

and S. Almazan-Cuellar, Ophthalmic Lenses measurement using Hartmann test, 5th 
Iberoamerican Meeting on Optics and 8th Latin American Meeting on Optics, Lasers, and Their 


[23] Optispheric and Optomatic by Trioptics, (Wedel, Germany) www.trioptics.com


[33] Robodyssey (Trenton, NJ) www.robodyssey.com (BX24 microcontroller and serial LCD)

Author Vitae

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