Photon Tunneling and Mode Selection due to the Absorptive Layers

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Abstract—Compared with the well-known electron coherent tunneling phenomenon, the photonic tunneling or mode selection mechanism in a structure with multiple absorptive layers are investigated. Due to the energy loss in the absorptive layers, the photon cannot pass through these barriers with 100% transmission as its electron counterpart. However, it is showed that some peaks appear in the transmission spectrum due to the coherent interference. It is also found that for one kind of transmission peaks, their optical fields form a standing wave pattern along the propagation direction that minimizes the optical loss. The possibility to apply this phenomenon in the optical filter and amplifier design is discussed.

Index Terms—Photon barrier, photon tunneling, and optical filter.

I. INTRODUCTION

Electron tunneling effect is a well known quantum phenomenon. Based on this interesting effect, some important devices were invented. They have shown great impacts on modern scientific and engineering applications, such as electron scanning tunneling microscope (STM) [1]. Coherent electron tunneling effect provides even more “unusual” results compared with the classical theory prediction. As shown in Figure. 1, when electron energy is less than the barrier height, at certain resonant energy level, the electron can totally tunneling through the barriers achieving 100% transmission. It is easily understood that resonant tunneling effects should lead to some novel devices. For examples, based on this principle, the resonant tunneling diode is a semiconductor device with potential of very fast speed modulation [2]; and inter-subband quantum cascade laser diode also attracts a lot of attentions due to its high efficiency [3].

\[ \Psi = A e^{i \beta x} + B e^{-i \beta x} \quad \text{in the well region} \]
\[ \Psi = Ce^{-\alpha x} + De^{\alpha x} \quad \text{in the barrier region} \]

Here A, B, C and D are arbitrary parameters determined by the
boundary conditions. $\beta$ is the electron wave vector defined by $\beta = (2mE)^{1/2} / h$, where $m$ is the electron effective mass, $E$ is its energy and $h$ is the Planck constant. $\alpha$ looks like an absorption coefficient in the barrier region defined by $\alpha = [2m(E - V)]^{1/2} / h$, where $V$ is the potential energy of the barrier. Since in the barrier region, there is no loss mechanism, electron probability function is still conserved. Therefore, the coefficient $\alpha$ only indicates the decay of the electron wave, but no electron absorption occurs in this region. This explains that in coherent tunneling case shown in Figure 1, the sum of reflectivity and transmission equals to 1 and coherent transmission can reach 100%. Our previous work on photon tunneling also keeps this conservation law \cite{12}. However, it is noticed that mathematically, coefficient $\alpha$ looks like an absorption coefficient of a lossy layer for the electromagnetic wave. In this work, we try to demonstrate the photon tunneling or more strictly speaking, mode selection phenomenon of a multiple absorptive layers structure.

In this paper, first, in section 2, we will present the basic theory to explain the photonic barrier caused by the absorptive layer in a plane wave approach. In section 3, we will demonstrate the photon coherent tunneling for the double-barrier case. To show the engineering design of a photonic tunneling filter, in section 4, a multi-layers tunneling filter will be introduced and the key design parameter is investigated. Finally we will give a brief conclusion.

II. THEORY OF PLANE WAVE TUNNELING

Not losing the generality, we assume the plane wave incidents on a two identical absorptive layers structure as shown in Figure 2. The polarization of E field is along x direction and propagation direction is z. Therefore the H field polarization is along the y direction. It is denoted that

$$E = \text{Re}(\tilde{E}e^{i\omega t})$$

$$H = \text{Re}(\tilde{H}e^{i\omega t}) .$$

(2)

Here $\omega$ is the wave frequency. The E and H field in each section along the propagation direction can be written as

$$\tilde{E}_i^j(z) = \tilde{E}_i^j e^{-jk_i(z-z_j)} + \tilde{E}_r^j e^{jk_i(z-z_j)}$$

$$\tilde{H}_i^j(z) = \frac{\tilde{E}_i^j}{\eta_i} e^{-jk_i(z-z_j)} - \frac{\tilde{E}_r^j}{\eta_i} e^{jk_i(z-z_j)} .$$

(3)

Here i denotes the section index. The subscripts F and R denote the forward and backward wave respectively. $k$ is a general complex wave vector and in the absorptive layer it is a complex number expressed as $k_i = \beta_i - j\alpha_i$. $\eta$ is the intrinsic impedance, which is a complex number in the absorptive layer, too, defined as $\eta = (\mu/\varepsilon_c)^{1/2}$. $\mu$ is the permeability and $\varepsilon_c$ is the complex permittivity defined by $\varepsilon_c = \varepsilon - je^\omega$.

Therefore generally, the wave vector can be calculated as $k^2 = \omega^2 \mu \varepsilon$. If the loss mechanism is due to the Ohm heat, we have $e^- = \sigma / \omega$. $\sigma$ is the conductivity of the loss layer. Here it is noted that for electromagnetic wave, wave vector is always a complex number with a real number in the transparent layer as the special case. However it can never be a pure imaginary number as the electron in the barrier region. In other words, while the wave decays, it also propagates unless the refractive index equals to zero, which is impossible practically. This causes the main difference between our cases with the electron counterpart. From the boundary condition of $E$ and $H$ fields, we can get the following transfer matrix

$$\begin{pmatrix}
\tilde{E}^{i+1}_F \\
\tilde{E}^{i+1}_R
\end{pmatrix} =
\begin{pmatrix}
m_{11} & m_{12} \\
m_{21} & m_{22}
\end{pmatrix}
\begin{pmatrix}
\tilde{E}^i_F \\
\tilde{E}^i_R
\end{pmatrix} .$$

(4)

The matrix elements are defined as

$$m_{11} = \frac{e^{-jk_i(z_{i+1} - z_j)}}{2} (1 + \eta_{i+1})$$

$$m_{12} = \frac{e^{jk_i(z_{i+1} - z_j)}}{2} (1 - \eta_{i+1})$$

$$m_{21} = \frac{e^{-jk_i(z_{i+1} - z_j)}}{2} (1 - \eta_{i+1})$$

$$m_{22} = \frac{e^{jk_i(z_{i+1} - z_j)}}{2} (1 + \eta_{i+1}) .$$

(5)

It is obvious that these equations are general and can be easily extended to an arbitrary number of layers application. If the total transfer matrix is $T_r$, then in our case, we have

$$T_r = M_4^1 M_3^2 M_2 M_1 .$$

(6)

The output field is related to the input field by
transmission is similar to the equivalent single barrier case. Therefore, we conclude that it is a mode selection process, and the optical fields of other wavelengths without coherence with the well region modes will be greatly suppressed. It is noted that the selected wavelengths agree with the cavity modes of the well region. It is also interesting to observe the reflection spectrum. We found that due to the energy loss, the sum of reflection and transmission is not equal to one any more as the electron tunneling phenomenon. Here we can see actually the reflection peaks’ wavelengths are coincident with those of transmission. We present the reflectivity spectrum in Figure 4.

Fig. 4. Reflectivity spectrum for the double barriers each 20 nm long and well length between them equal to 6 μm.

In Figure 5, we show the field density for the above phenomenon and focus to the region with two long wavelength peaks around 6000 nm and 12000 nm. It is found that there are two dark channels around these wavelengths along the wave propagation direction. The other wavelength show stronger density and therefore causes more loss while propagating.

Fig. 5. Field density distribution contour along the wave propagation direction. There are two dark channels along the z direction corresponding to the two tunneling peaks shown in Fig. 3.

There is one case of great practical application interests. If the barrier width is the same as that of the well, the propagating wave sets up a standing wave pattern as shown in Figure 6.

Here the absorption coefficient is wavelength dependent, at shorter wavelength or high frequency region the absorption is very strong. At longer wavelength region after 2500 nm, the absorption is smaller and almost independent of the wavelength. We found that there is a mode selection or tunneling mechanism for the double barriers case. However, it is quite different from the electron due to the loss layer. Instead of reaching 100% transmission, here the maximum
As shown in Figure 6, in this case, the standing wave valleys fall into the loss layers and its peak in the transparent well region. Therefore the propagation loss is at the minimum in this configuration. This mode will be selected and show a peak in the transmission spectrum. We present the transmission spectrum of above case in Figure 7 with barrier and well width both equal to 200 nm. Here the conductivity in the loss layer is $5.8 \times 10^3 \text{ m}^{-1} \Omega^{-1}$.

Here we can see a transmission peak around 800 nm, which is corresponding to the standing wave in a 200 nm absorptive layer and a 200 nm well.

Here we only consider the TE mode. The optical field propagating in the waveguide can be expressed as \[ \tilde{E}(X, z) = [\tilde{E}_F(z) e^{-j \frac{\pi}{\Lambda} x} + \tilde{E}_R(z) e^{j \frac{\pi}{\Lambda} x}] \Phi(X) \quad , \tag{9} \]

where $X$ is the cross sectional coordinates ($x, y$), and $z$ is the longitudinal direction along the waveguide. $\Phi(X)$ is the cross sectional optical field profile function. Here the waveguide has been designed to support only one single cross sectional mode. The wave equation is

\[ \nabla^2 \tilde{E} + \epsilon_c(X, z) \frac{\omega^2}{c^2} \tilde{E} = 0 \quad , \tag{10} \]

Due to the grating structure, we have written the permittivity as

\[ \epsilon_c(X, z) = \begin{cases} 1 & \text{for absorptive layer} \\ 1 & \text{for transparent layer} \end{cases} \]

IV. CASCADE LOSS TUNNELING

In this section, we apply the standing wave tunneling phenomenon presented previously to a cascade multi-wells realized in the waveguide similar to a purely loss coupled DFB laser [13]. The proposed structure is schematically shown in Fig. 8.

Here the center guided region is sandwiched by the lower refractive index material and the waveguide shows a ridge structure such that cross section field profile is confined in the guided region. In the guided layer, a gratings structure is formed with loss in the green area and transparency in the blue area. However the refractive index of these two parts is the same. This is similar to what we showed in the previous section except that the loss layers and wells are cascaded to form a grating structure. We denote the period of the grating as $\Lambda$. In practice, it is not easy to realize such a structure without index contrast in the grating. Recently, we found that it is possible to fabricate the purely loss coupled grating by applied the coherent control effects in an asymmetric quantum well material [14]. The realization is beyond the scope of this work, we direct the readers to the references [14, 15]. We also assume the waveguide can be pumped independently to get a uniform gain along the cavity electrically or optically.

In this section, we apply the standing wave tunneling phenomenon presented previously to a cascade multi-wells realized in the waveguide similar to a purely loss coupled DFB laser [13]. The proposed structure is schematically shown in Fig. 8.

Here we can see a transmission peak around 800 nm, which is corresponding to the standing wave in a 200 nm absorptive layer and a 200 nm well.
\[
\varepsilon_c(X,z) = \varepsilon_B(X,z) + \Delta \varepsilon(X,z)
\]

\[
\Delta \varepsilon(X,z) = \sum_m \Delta \varepsilon_{zm}(X) e^{-j m \pi z / \Lambda}.
\]  

Here \( \varepsilon_B \) is the average background permittivity, and \( \Delta \varepsilon \) is the variation caused by the loss contrast of the grating. Substituting Eqns. (9) and (11) into Eqn. (10), we obtain the cross sectional optical mode equation

\[
\nabla^2_{X} \Phi(X) + [\varepsilon_B(X) \frac{\omega^2}{c^2} - (\frac{\omega}{c} n_{\text{eff}})^2] \Phi(X) = 0.
\]

Here \( n_{\text{eff}} \) is the effective refractive index. Solving this 2D eigenfunction problem, and using the perturbation theory \([16]\), if only the first order coupling is considered, finally we obtain the well-known coupled-mode equations \([16,18]\)

\[
\frac{d\vec{E}_F}{dz} = (G - j \delta) \vec{E}_F - jk_+ \vec{E}_R
\]

\[
\frac{d\vec{E}_R}{dz} = -(G - j \delta) \vec{E}_R + jk_- \vec{E}_F.
\]

In Eqn. 13, \( G \) is the net modal gain/loss, which is controllable due to the waveguide material pumping or coherent control mechanism as mentioned before. \( \delta = (\omega / c) n_{\text{eff}} - \pi / \Lambda \) is called detuning factor and \( k_{\pm} \) is the coupling coefficient defined by

\[
k_{\pm} = \frac{(\omega / c) [\Delta \varepsilon_{\pm 1}(X) |\Phi(X)|^2 dX}{2n_{\text{eff}} \int |\Phi(X)|^2 dX}.
\]

Here, we only consider the first order grating case corresponding to \( m = \pm 1 \) in Eqn. 14. It is found that because the perturbation is coming from loss contrast only, the coupling coefficient from Eqn. 14 is always a pure imaginary number. Actually, it can be written as \( k_+ = j k_L, k_- \) is a negative number.

In the following calculation, we assume the waveguide is fabricated by AlGaAs/InAlAs/GaAs material, and the wavelength is infrared, whose gain/loss can be coherently controlled without introducing refractive index variation \([14]\). Here the effective refractive index of the waveguide is \( n_{\text{eff}} = 3.20 \). The grating period \( \Lambda = 0.7764 \mu m \) such that the wavelength corresponding to the standing wave pattern shown in Figure 6 is around 4969 nm, which is also called Bragg wavelength. The loss coupling coefficient due to the loss contrast \( k_L = -3.33 \times 10^{-3} \mu m^{-1} \). The pumping mechanism can provide \( G = 20 \text{ cm}^{-1} \) uniformly along the waveguide. In Figure 9, we show as the length of the waveguide gets longer, i.e., more well/barrier structures are cascaded, the mode selection mechanism gets stronger with the selected mode greatly enhanced and side modes greatly suppressed. This can be applied as an optical filter and amplifier.

V. CONCLUSIONS

In this paper, we explained the coherent photon tunneling effect realized by the loss layers as barriers and the transparent part between two barriers as the photon well. For this configuration, the difference between photon and electron is phenomenal due to the fact that absorptive layer of photon is a real lossy medium, but for electron it is just a decay layer mathematically. Therefore for loss layer as barrier, the photon transmission can never reach 100\%, but the resonant modes are selected while photons pass through the double barriers and well structure. One kind of mode selection is corresponding to the photons in resonant with the well cavity modes; the other is due to the standing wave pattern. We also demonstrated that for the latter case, it is possible to fabricate a cascade tunneling structure along the waveguide similar to a
purely loss coupled DFB laser diode. It is shown that if some pumping mechanisms are applied to provide a uniform gain along the waveguide. This device may be used as an optical filter and amplifier.

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