MOTOR VEHICLE HANDLING AND STABILITY PREDICTION

Stan A. Lukowski

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ABSTRACT

The directional response and stability characteristics of the motor vehicle are examined by considering the automobile to be a mechanical system which is described by linear differential equations with constant coefficients. The directional response and stability characteristics of the motor vehicle are important performance modes of vehicle operation often equated with handling. The objective of this analysis is to show how steady-state turning behavior depends upon various vehicle design factors and motion variables. A family of characteristic handling diagrams are obtained for linear and non-linear range of tire operation. The handling diagrams show the dependence of vehicle directional behavior upon tire lateral force characteristics, front/rear load distribution and vehicle forward speed. Information, obtained from this study of vehicle response to steering control, gives insight into turning behavior of a real vehicle and is useful to safety objectives.
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1. INTRODUCTION

The purpose of this study is to develop analytical methods that are useful for examining or predicting the dynamic behavior of a motor vehicle subjected to control inputs. The term “dynamic behavior” should be understood as vehicle performance, that is, the lateral-directional response behavior exhibited by the motor vehicle subjected to any control inputs or external disturbances, either singly or in combination.

The directional response behavior and stability characteristics of the motor vehicle are important performance modes of vehicle operation often equated with handling. Handling is a loosely used term meant to imply the responsiveness of a vehicle to driver input, or the ease of control. As such, handling is an overall measure of the vehicle-driver combination. The driver and vehicle form a closed-loop system — meaning that the operation of an automobile involves interaction between vehicle, driver and the road. For purposes of characterizing only the vehicle, open-loop behavior is considered here in this study. Open-loop refers to vehicle response to specific steering inputs, and is more precisely defined as directional response behavior [1]. A simulation model specifically design to study vehicle directional response to control inputs is developed and executed on a digital computer. This report concludes with the presentation of computer simulation findings with respect to the motor vehicle system handling qualities and stability characteristics as influenced by service factors, that is, by tire lateral force characteristics and the front/rear load distribution.
2. VEHICLE MODEL AND EQUATIONS OF MOTION

2.1. Vehicle Model Description

In this study, attention is focused on more fundamental aspects of vehicle dynamic behavior. For this purpose, a simplified one-mass vehicle model that is free to yaw and sideslip while negotiating a turn with a constant speed is examined. The vehicle model as shown in Fig. 1 does not have body roll and load transfer. The driving torque applied to the drive wheels that is required to keep the vehicle speed constant is assumed to be small, permitting all wheels to be treated as “free rolling” such that the tire lateral force depends only on the tire lateral slip. As a result of small disturbance assumptions (made for the purposes of linearization) it can be concluded that the variation in longitudinal forces during directional motion is negligible. Consequently, the vehicle does not experience any longitudinal accelerations, provided that the driving torque is in equilibrium with the resistance to forward motion. The resistance develops primarily from the aerodynamic drag and rolling resistance forces produced by the tires. The control inputs to the vehicle model consist of relatively small steering wheel angular displacements. Thus, small steering inputs do not cause any change in vehicle forward velocity with the result that linearized treatment of vehicle motion analysis implies constant velocity maneuvers.

The above limitations concerning the vehicle model are necessary for the analysis to remain relatively simple and easy to comprehend. These limitations will also allow us to determine the primary factors influencing vehicle directional behavior.
2.2. Formulation of Governing Equations

In order to derive the governing equations of motion of the vehicle model, we can either employ the Newton-Euler approach or the methods of analytical mechanics. In this case, it proves to be particularly convenient to use the latter procedure, namely Lagrange’s equations. However, a minor complication results from the fact that customary Lagrange equations written in terms of generalized coordinates only yield meaningful results when the generalized coordinates are also inertial or true coordinates. Mathematically, a coordinate system may be considered “true” if integration of the body’s velocity vectors with respect to time yields the corresponding location coordinates.

Unfortunately, expression of vehicle motion in terms of a fixed, inertial coordinate system when the vehicle is undergoing simultaneous translation, yaw, and/or roll motions is very cumbersome. To circumvent this difficulty, the vehicle motions can be expressed in terms of a moving coordinate system which translates and yaws with the vehicle. However, these moving coordinates will not be inertial, and thus will not satisfy the integrability requirement of true
coordinates. Therefore, application of customary Lagrange’s equations to a moving coordinate system will result in erroneous equations of motion.

A method does exist, however, by which Lagrange’s equations may be correctly derived for a moving coordinate system. This method requires the use of so-called quasi-coordinates, and the Lagrange’s equations expressed in terms of these coordinates are called the Special Lagrange Equations [2, 3].

### 2.3. Special Lagrange Equations

For the model of the vehicle being considered, the Special Lagrange Equations have the following form:

\[
\frac{d}{dt} \left( \frac{dT}{du} \right) - r \left( \frac{dT}{dv} \right) = \sum F_x
\]

\[
\frac{d}{dt} \left( \frac{dT}{du} \right) + r \left( \frac{dT}{dv} \right) = \sum F_y
\]

\[
\frac{d}{dt} \left( \frac{dT}{dr} \right) - v \left( \frac{dT}{du} \right) + u \left( \frac{dT}{dv} \right) = \sum M_z
\]

A detailed derivation of these equations can be found in Reference [3].

The total kinetic energy of the vehicle model may be written in terms for the translational and rotational (yawing) velocities, such that

\[
T = \frac{1}{2} mV^2 + \frac{1}{2} I_{zz}r^2
\]

The translational velocity, \( V \), consists of \( u \) and \( v \) components and may be written as

\[
v^2 = u^2 + v^2
\]

Substituting for the translational velocity from Eq. (5) into Eq. (4), the total kinetic energy of the considered vehicle model may then be written as

\[
T = \frac{1}{2} m(u^2 + v^2) + \frac{1}{2} I_{zz}r^2
\]
Evaluating the terms in Eqs. (1), (2) and (3), yields

\[ \frac{dT}{du} = mu \tag{7} \]

\[ \frac{d}{dt} \left( \frac{\partial T}{\partial u} \right) = m\ddot{u} \tag{8} \]

\[ \frac{dT}{dv} = mv \tag{9} \]

\[ \frac{d}{dt} \left( \frac{\partial T}{\partial v} \right) = m\dot{v} \tag{10} \]

\[ \frac{dT}{dr} = I_{zz} r \tag{11} \]

\[ \frac{d}{dt} \left( \frac{\partial T}{\partial r} \right) = I_{zz} \dot{r} \tag{12} \]

Substituting the appropriate partial derivatives from Eqs. (7) through (12) into Eqs. (1) through (3) yields three differential equations of motion

\[ m(\ddot{u} - rv) = \sum F_x \tag{13} \]

\[ m(\ddot{v} - ru) = \sum F_y \tag{14} \]

\[ I_{zz} \dot{r} = \sum M_z \tag{15} \]

Equations (13), (14) and (15) form a set of three differential equations which describe the motion of a vehicle with longitudinal, lateral and yaw degrees of freedom.

### 2.4. Modified Equations of Vehicle Motion

Since the yaw and sideslip velocities are known to be small compared to the steady-state forward velocity of the vehicle, the products of these terms (i.e., $rv$) may be considered negligible. Based on these assumptions, we obtain the following three coupled differential equations of motion to be employed in this analysis.

\[ \sum F_x = m\dot{u} \tag{16} \]

\[ \sum F_y = m(\dot{v} + ru) \tag{17} \]

\[ \sum M_z = I_{zz} \dot{r} \tag{18} \]
Note that a longitudinal acceleration term, \( \dot{u} \), appears only in the first equation of the above three equations of motion. If we assume that forward speed \( u \) is kept constant, then \( \dot{u} = 0 \).

Consequently, we can ignore the first equation and conclude that the remaining two equations apply to a one-mass motor vehicle which is moving with a constant velocity, \( u \), along its longitudinal axis. Thus, the linearized equations of motion for the assumed vehicle model are

\[
\sum F_y = m(\dot{v} + ru) \quad (19)
\]
\[
\sum M_z = I_{zz}\dot{r} \quad (20)
\]
3. VEHICLE HANDLING AND STABILITY UNDER STEADY-STATE CONDITIONS

3.1. Derivation of the Handling Diagram for the Linear Range of Tire Operation

For steady-state, we may further assume that the lateral and yawing accelerations are zero, that is, \( \dot{\nu} = \dot{\phi} = 0 \). Additionally, for a small side slip angle, it can be assumed that \( u \equiv V \). Under these assumptions, Eqs. (19) and (20) become

\[
\sum F_y = m\nu r \quad (21)
\]
\[
\sum M_z = 0 \quad (22)
\]

where

\[
\sum F_y = F_{y1} + F_{y2} + F_{y3} + F_{y4} \quad (23)
\]
\[
\sum M_z = a(F_{y1} + F_{y2}) - b(F_{y3} + F_{y4}) \quad (24)
\]

The tires’ lateral force components along the vehicle axes are computed from

\[
\sum F_{yi} = F_{ywli} \cos (\delta_i) \quad (25)
\]

where the steering angles \( \delta_i \ (i = 1, 2, 3, 4) \) are

\[
\delta_1 = \delta_2 = \delta \quad (26)
\]
\[
\delta_3 = \delta_4 = \delta \quad (27)
\]

For small front wheel steer displacement, \( \delta \), it can be assumed that \( \cos (\delta) \approx 1 \). Thus Eq. (25) becomes

\[
\sum F_{yi} = F_{ywli} \quad (28)
\]
Note that each tire on the vehicle is side-slipping and turning on a curved path of radius R. Ignoring the lateral distortion of the tire due to path curvature and assuming that the slip angles remain small such that the cornering force is linearly related to the slip angle (see Fig. 2), we may write that for a linear range of tire operation, the lateral tire forces generated at the tire/road interface oriented with respect to the wheel plane can be computed from

$$ F_{ywi} = -\left( \frac{\partial F_{yi}}{\partial \alpha_i} \right) \alpha_i $$

(29)

where

$$ \frac{\partial F_{yi}}{\partial \alpha_i} \rightarrow \text{is the cornering stiffness of the } i^{th} \text{ tire (i=1, 2, 3, 4)} $$

and

$$ \alpha_i \rightarrow \text{is the slip angle of the } i^{th} \text{ tire} $$

In a steady turn ($u$, $v$, and $r$ are fixed quantities) the lateral velocities at the front and rear axles, $v_1$ and $v_2$, are given by (see Fig. 1)

$$ v_1 = v + ar $$

(30)

$$ v_2 = v - br $$

(31)
In this instance, we find that the angles between the velocity vector of each tire and the tire center-plane are as diagrammed in Fig 1. Note that if \( u \gg v + ar \), \( u \gg v - br \), and \( \beta \) is a small angle, the front and rear slip angles may be expressed as
\[
\alpha_f = \alpha_{1,2} = \frac{v}{u} + \frac{ar}{u} - \delta = \beta + \frac{ar}{u} - \delta \tag{32}
\]
\[
\alpha_r = \alpha_{3,4} = \frac{v}{u} + \frac{br}{u} = \beta - \frac{br}{u} \tag{33}
\]
Denoting \( \frac{\partial F_y}{\partial \alpha} \) \(_f\) and \( \frac{\partial F_y}{\partial \alpha} \) \(_r\) as the resultant lateral stiffness of both front and both rear tires, respectively, we may write that
\[
F_{yw1,2} = -\left( \frac{\partial F_y}{\partial \alpha} \right) \_f \alpha_f \tag{34}
\]
\[
F_{yw3,4} = -\left( \frac{\partial F_y}{\partial \alpha} \right) \_r \alpha_r \tag{35}
\]
With the aid of Eqs. (30) through (35), the following equations of equilibrium are obtained
\[
\begin{align*}
\left[ \frac{\partial F_y}{\partial \alpha} \right] \_f + \left[ \frac{\partial F_y}{\partial \alpha} \right] \_r \beta + \left[ \frac{a \cdot \frac{\partial F_y}{\partial \alpha} \_f}{u \cdot \frac{\partial \alpha}{\partial \alpha}} - \frac{b \cdot \frac{\partial F_y}{\partial \alpha} \_r}{u \cdot \frac{\partial \alpha}{\partial \alpha}} \right] + mV \right] r = \left[ \frac{\partial F_y}{\partial \alpha} \right] \_f \delta \\
\left[ a \cdot \frac{\partial F_y}{\partial \alpha} \_f - b \cdot \frac{\partial F_y}{\partial \alpha} \_r \right] \beta + \left[ \frac{a^2 \cdot \frac{\partial F_y}{\partial \alpha} \_f}{u \cdot \frac{\partial \alpha}{\partial \alpha}} + \frac{b^2 \cdot \partial F_y}{\partial \alpha} \_r \right] r = \left[ a \cdot \frac{\partial F_y}{\partial \alpha} \_f \delta \right]
\end{align*}
\]
Eqs. (36) and (37) can be expressed in matrix form as
\[
\begin{pmatrix}
\frac{\partial F_y}{\partial \alpha} \_f + \frac{\partial F_y}{\partial \alpha} \_r & a \cdot \frac{\partial F_y}{\partial \alpha} \_f - b \cdot \frac{\partial F_y}{\partial \alpha} \_r & \frac{a^2 \cdot \frac{\partial F_y}{\partial \alpha} \_f}{u \cdot \frac{\partial \alpha}{\partial \alpha}} + \frac{b^2 \cdot \partial F_y}{\partial \alpha} \_r \\
\end{pmatrix}
\begin{pmatrix}
\beta \\
\end{pmatrix}
= \begin{pmatrix}
\frac{\partial F_y}{\partial \alpha} \_f \\
\end{pmatrix}
\delta
\]
Solving the above equation for the yawing velocity, \( r \), with the aid of Cramer’s rule, we obtain
\[
\begin{pmatrix}
\frac{\partial F_y}{\partial \alpha} \_f + \frac{\partial F_y}{\partial \alpha} \_r & a \cdot \frac{\partial F_y}{\partial \alpha} \_f - b \cdot \frac{\partial F_y}{\partial \alpha} \_r & \frac{a^2 \cdot \frac{\partial F_y}{\partial \alpha} \_f}{u \cdot \frac{\partial \alpha}{\partial \alpha}} + \frac{b^2 \cdot \partial F_y}{\partial \alpha} \_r \\
\end{pmatrix}^{-1}
\begin{pmatrix}
\frac{\partial F_y}{\partial \alpha} \_f + \frac{\partial F_y}{\partial \alpha} \_r & \frac{\partial F_y}{\partial \alpha} \_f \\
\end{pmatrix}
\delta
\]
\[
\left( \frac{\partial F_y}{\partial \alpha} \_f - b \cdot \frac{\partial F_y}{\partial \alpha} \_r \right) a \cdot \frac{\partial F_y}{\partial \alpha} \_f - b \cdot \frac{\partial F_y}{\partial \alpha} \_r \\
\end{pmatrix} \frac{\partial F_y}{\partial \alpha} \_f \delta
\]
On evaluating Eq. (39) for the yawing velocity, \( r \), we obtain the following expression for the ratio of the steady yaw rate to the front-wheel steer displacement, \( \delta \):
\[ r \delta = \frac{\left[ \frac{\partial F_y}{\partial \alpha} \right]_f + \frac{\partial F_y}{\partial \alpha} \right]_r}{\left[ \frac{\partial F_y}{\partial \alpha} \right]_f + \frac{\partial F_y}{\partial \alpha} \right]_r} \left[ \frac{a \cdot \frac{\partial F_y}{\partial \alpha} - b \cdot \frac{\partial F_y}{\partial \alpha} \right]_f - \left[ \frac{a \cdot \frac{\partial F_y}{\partial \alpha} - b \cdot \frac{\partial F_y}{\partial \alpha} \right]_r \right] - D \]

where

\[ D = \left[ a \cdot \frac{\partial F_y}{\partial \alpha} - b \cdot \frac{\partial F_y}{\partial \alpha} \right]_f - \left[ \frac{a}{u} \cdot \frac{\partial F_y}{\partial \alpha} - b \cdot \frac{\partial F_y}{\partial \alpha} \right]_r + mV \]

The ratio of \( r/\delta \) is called the yaw rate gain. Since \( V = r\bar{R} \), the yaw rate gain can be transformed to a path curvature gain by noting that

\[ \frac{1}{\bar{R}} = \frac{1}{V} \frac{r}{\delta} \]

Also noting that \( u \approx V \) and with the aid of Eq. (41), Eq. (40) is transformed to the following relation

\[ \frac{1}{\bar{R}} = \frac{[a + b] \cdot \frac{\partial F_y}{\partial \alpha}_f \cdot a \cdot \frac{\partial F_y}{\partial \alpha}_r}{[a + b]^2 \cdot \frac{\partial F_y}{\partial \alpha}_f \cdot \frac{\partial F_y}{\partial \alpha}_r - mV^2 \left[ \frac{a \cdot \frac{\partial F_y}{\partial \alpha}_f - b \cdot \frac{\partial F_y}{\partial \alpha}_r \right]} \]

Dividing the right-hand side terms of Eq. (42) by the numerator and substituting for \((a+b)=l\), the expression for the path curvature gain reduces to

\[ \frac{1}{\bar{R}} = \frac{1}{1 - mV^2 \left( \frac{YM}{l^2 CY} \right)} \]

where the term

\[ YM = a \cdot \frac{\partial F_y}{\partial \alpha}_f - b \cdot \frac{\partial F_y}{\partial \alpha}_r \]

is called in this analysis the yawing moment coefficient and

\[ CY = \frac{\partial F_y}{\partial \alpha}_f \cdot \frac{\partial F_y}{\partial \alpha}_r \]

is the product of the front and rear tires’ cornering stiffnesses.
Inversely, Eq. (43) can be written as
\[
\delta = \frac{l}{R} \left[ 1 - mV^2 \left( \frac{Y M}{l^2 C Y} \right) \right]
\] (46)

Or, the steer angle required to travel in a steady turn of radius \( R \) is
\[
\delta = \frac{l}{R} \left[ 1 - mV^2 \left( \frac{Y M}{l^2 C Y} \right) \right]
\] (47)

Eq. (46) gives the variation in steer angle, \( \delta \), that is required for a given vehicle to negotiate a turn of fixed radius \( \bar{R} \). Fig. 3 presents the ratio \( \frac{\delta}{(l/R)} \) plotted against vehicle velocity \( V^2 \).

The diagram of Fig. 3 shows the variation in steer angle that is required for the vehicle to move in a constant radius curve when the speed is increased. Three different kinds of vehicles are considered: *an oversteer (YM>0), a neutral steer (YM=0)* and *an understeer (YM<0)*.

For an understeer vehicle, the required steer angle increases with increasing speed. The figure indicates that for an oversteer vehicle the required steer angle diminishes with increasing speed. In the latter case, the required steer angle changes sign at a speed termed the *critical speed*. The expression for the critical speed can be obtained by noting that the ratio \( \frac{\delta}{(l/R)} \) at this speed equals zero. Thus, equating the left-hand term of Eq. (46) to zero and solving for \( V_{cr} \), gives
\[
V_{cr} = l \frac{C Y}{m Y M}
\] (48)

where YM and CY are defined by Eqs. (44) and (45), respectively.
Let us again consider Eq. (46) which can also be expressed as

\[ \delta = \frac{l}{R} - \frac{l}{R} m V^2 \left( \frac{YM}{l^2 CY} \right) \]  

or as

\[ \frac{l}{R} - \delta = \frac{WYM}{CY} \left( \frac{V^2}{gR} \right) \]  

Based on Eqs. (32 and (33), we find that the difference in front and rear slip angles is given by

\[ \alpha_f - \alpha_r = \frac{r}{u} l - \delta \]  

For small side slip angle, the path curvature in a steady turn is given by \( \frac{1}{R} = \frac{r}{v} \approx \frac{r}{u} \). The substitution of \( \frac{r}{u} = \frac{1}{R} \) into Eq. (51), yields

\[ \alpha_f - \alpha_r = \frac{l}{R} - \delta \]  

Thus, the vehicle force and moment balance for a given steer angle (i.e., Eq. (50)) can be related to the difference in front and rear slip angles as given by Eq. (52) and the diagram of Fig. 3 can be transformed to that of Fig. 4. In that figure, the new ordinate represents the difference in front
and rear slip angles ($\alpha_f - \alpha_r$). The new abscissa represents the centripetal acceleration ($V^2/g\cdot R$) in units of acceleration of gravity, $g$. The handling diagram that we wish to derive is obtained by rotating the plot of Fig. 4 through 90° and combining it with a diagram for centripetal acceleration against path curvature, $l/R$. The space of the latter diagram can be filled with lines of constant velocity, lines of constant radius and lines of constant steer angle.

Fig. 4. Difference of Slip Angles vs. Lateral Acceleration

Fig. 5 represents the combined diagrams for three vehicles with the same mass and wheelbase, but with different handling characteristics. The figure shows that for a given handling characteristic (thick line) the steer angle, $\delta$, required for negotiating a certain maneuver characterized by $R$ and $V$ can be read directly from the diagram. In the linear range of the handling regime, the steer angle $\delta$ changes linearly with lateral acceleration at a given radius.

When the yawing moment coefficient is negative, i.e., $YM<0$, the steer angle increases with lateral acceleration (i.e., with an increase of speed for a constant radius test), and the vehicle is described as **understeering**. For $YM>0$, the vehicle is described as **oversteering**. For $YM=0$, the vehicle is said to exhibit **neutral steer**.
3.2. Derivation of the Handling Diagram for the Nonlinear Range of Tire Operation

The steady-state turning behavior of the vehicle discussed so far is limited to the case when tires operate at a small slip angle. As shown in Fig. 2, at small slip angles, the relationship between lateral forces $F_y$ and slip angle $\alpha$ is linear. At larger slip angles, the tire operates in the nonlinear range. When we extend the vehicle model to include nonlinear characteristics, Eq. (49) can no longer be used to relate steer angle $\delta$ to the force and moment balance on the vehicle. We shall now develop a graphical method for obtaining a handling diagram for the nonlinear range of tire operation. The graphical method presented in this section is based on work performed by Pacejka [4].

Assuming a fixed set of tire characteristics which do not change during the motion, Eqs. (21) and (22) can be expressed as

$$\sum F_y = mV^2 \left( \frac{1}{R} \right)$$

$$aF_{yf} = bF_{yr}$$

Fig. 5. Handling Diagram for the Linear Range of Tire Operation
where

\[ \sum F_y = F_{yf} + F_{yr} \]

\[ F_{yf} = F_{y1} + F_{y2} \]

\[ F_{yr} = F_{y3} + F_{y4} \quad \text{and} \]

\[ \frac{1}{R} = \frac{r}{\bar{V}} \]

Solving Eqs. (53) and (54) for \( F_{yf} \) and \( F_{yr} \) we obtain

\[ F_{yf} = mV^2 \left( \frac{1}{R} \right) \left( \frac{b}{l} \right) \quad (55) \]

\[ F_{yr} = mV^2 \left( \frac{1}{R} \right) \left( \frac{a}{l} \right) \quad (56) \]

From Eqs. (55) and (56) we obtain

\[ mV^2 \left( \frac{1}{R} \right) = \frac{F_{yf}}{b/l} = \frac{F_{yr}}{a/l} \quad (57) \]

Normalizing the relationships given by Eq. (57) with respect to the vehicle weight \( mg \), yields

\[ \frac{V^2}{gR} = \frac{F_{yf}}{mg(b/l)} = \frac{F_{yr}}{mg(a/l)} \quad (58) \]

Under the assumption that there is no load transfer, we may observe that the terms \( mg(b/l) \) and \( mg(a/l) \) represent the resultant vertical loads \( F_{zf} \) and \( F_{yr} \) on the front and rear tires, respectively. The term \( V^2/gR \) is the lateral acceleration. Therefore, we may write

\[ \frac{V^2}{gr} = \frac{F_{yf}}{F_{zf}} = \frac{F_{yr}}{F_{zr}} = \frac{F_{ynf}}{F_{ynr}} = \frac{F_y}{F_z} \quad (59) \]

Fig. 6 shows the lateral force characteristics of pneumatic tires (front and rear) normalized with respect to normal loads which occur on a dry road surface. According to Eq. (59), these two tire curves may be merged into one diagram with the same ordinate \( V^2/gR \). For a certain Vehicle velocity \( V \), a front slip angle \( \alpha_f \) and a rear slip angle \( \alpha_r \) may be read from this diagram.

Subtracting the normalized tire characteristics from each other in the horizontal direction, we may construct a diagram with ordinate \( V^2/gR \) versus \((\alpha_r-\alpha_f)\). This produces the handling curve shown in Fig. 7 which relates the difference of the slip angles and the lateral acceleration. The slope of the handling curve changes from negative to positive. Negotiation of a circular maneuver at gradually increasing speed with such a vehicle requires first an increasing steer
angle $\delta$ and, beyond a certain $V^2/gR$ value, a gradual reduction of $\delta$. The vehicle appears to be in understeer at low lateral accelerations and oversteer at high lateral accelerations.

**Fig. 6.** Normalized Tire Lateral Force Characteristics

**Fig. 7.** Handling Diagram for the Nonlinear Range of Tire Operation
4. SUMMARY AND CONCLUSIONS

4.1. Summary of Results

An analytical study has been performed to investigate the directional response behavior of a motor vehicle in steady-state turning maneuvers. This important performance mode of vehicle operation is termed vehicle handling.

Based on this study, a family of characteristics handling diagrams for different tire lateral force characteristics were obtained. The primary handling regime, that is, for linear range of tire operation is the first stage and is adequately represented by linear relationships. The handling diagrams were obtained from the linearized vehicle model and show the dependence of the vehicle directional behavior upon lateral tire force characteristics, front/rear load distribution and vehicle forward speed. The primary factor which defines vehicle handling character is the value of the coefficient $YM$ which determines the location of the so-called neutral steer point. Locations forward of the center of gravity (c.g.), that is when $a<b$, and when the rear tires’ cornering stiffness is higher than the front tires’ cornering stiffness, that is when $\frac{\partial F_y}{\partial \alpha}_r > \frac{\partial F_y}{\partial \alpha}_f$, result in a negative value of $YM$. In this instance, the vehicle is defined as an understeer. Locations to the rear of the c.g. and front tires’ cornering stiffness higher than rear tires’ cornering stiffness, that is when $\frac{\partial F_y}{\partial \alpha}_f > \frac{\partial F_y}{\partial \alpha}_r$, result in a positive value of $YM$. In this case, the vehicle is said to be an oversteer. It is clear that vehicles with c.g. near the center of the wheelbase and with front rear cornering stiffnesses approximately equal have value of $YM=0$ and are characterized as neutral steer vehicles.
The secondary handling regime which deals with the nonlinearities of tire lateral force characteristics becomes more complex. In general, it cannot be represented by simple equations. A great variety of handling curves can result due to differences in the elastic and frictional properties of tires. We may conclude that differences in tire slip angle give sufficient information about the steering character of an automobile when large lateral accelerations are involved. Alternative definitions of under- and oversteer refer to the sign of the slope of the handling curve. Referring to the handling diagram presented in Fig. (7), we have the following equivalent definition for oversteer (OVE), neutralsteer (NEU) and understeer (UND):

OVE: \[ \frac{\partial \left( \frac{V^2}{gR} \right)}{\partial (\alpha_f - \alpha_r)} > 0 \]

NEU: \[ \frac{\partial \left( \frac{V^2}{gR} \right)}{\partial (\alpha_f - \alpha_r)} = 0 \]

UND: \[ \frac{\partial \left( \frac{V^2}{gR} \right)}{\partial (\alpha_f - \alpha_r)} < 0 \]

4.2. Concluding Remarks

A theoretical study has been performed to investigate road vehicle handling qualities. From this study, the following conclusions can be drawn:

1. Theory of vehicle directional behavior has been developed and can be applied with relative ease.

2. Computer simulations were performed to study the handling dynamics of a vehicle undergoing steering maneuvers.

3. The results provided a family of characteristic handling curves to show the dependence of the vehicle directional behavior upon various vehicle design factors and motion variables.

4. Information, obtained from the studies of vehicle-system response to steering control, gives insight into the turning behavior of a real vehicle and is useful to safety objectives.

5. To evaluate the accuracy of the result obtained from the simulation it will be necessary to conduct vehicle response tests. Experimental counterparts of the maneuvers performed on the computer are needed to formulate basic criteria for assessing vehicle control quality from a safety point of view.
## 5. NOMENCLATURE

### 1. VEHICLE PARAMETERS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Units</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>$m$</td>
<td>Distance between front axle and vehicle center of mass</td>
</tr>
<tr>
<td>$b$</td>
<td>$m$</td>
<td>Distance between rear axle and vehicle center of mass</td>
</tr>
<tr>
<td>$\frac{\partial F_y}{\partial \alpha}_f$</td>
<td>$N/\text{rad}$</td>
<td>Resultant lateral stiffness of front tires</td>
</tr>
<tr>
<td>$\frac{\partial F_y}{\partial \alpha}_r$</td>
<td>$N/\text{rad}$</td>
<td>Resultant lateral stiffness of rear tires</td>
</tr>
<tr>
<td>$I_{zz}$</td>
<td>$\text{kg} \cdot \text{m}^2$</td>
<td>Vehicle yaw moment of inertia</td>
</tr>
<tr>
<td>$l$</td>
<td>$m$</td>
<td>Vehicle wheelbase</td>
</tr>
<tr>
<td>$m$</td>
<td>$\text{kg}$</td>
<td>Vehicle mass</td>
</tr>
<tr>
<td>$W$</td>
<td>N</td>
<td>Vehicle weight</td>
</tr>
</tbody>
</table>

### 2. VEHICLE VARIABLES

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{yi}$</td>
<td>Tire force along lateral body axis</td>
</tr>
<tr>
<td>$F_{ywi}$</td>
<td>Tire lateral force normal to the wheel plane</td>
</tr>
<tr>
<td>$F_{yf}$</td>
<td>Resultant lateral force produced by front tires</td>
</tr>
<tr>
<td>$F_{yr}$</td>
<td>Resultant lateral force produced by rear tires</td>
</tr>
<tr>
<td>$YM$</td>
<td>Yawing moment coefficient</td>
</tr>
<tr>
<td>$r$</td>
<td>Yawing velocity</td>
</tr>
<tr>
<td>$u$</td>
<td>Forward velocity of vehicle mass center</td>
</tr>
<tr>
<td>$v$</td>
<td>Lateral velocity of vehicle mass center</td>
</tr>
<tr>
<td>$V$</td>
<td>Resultant velocity of vehicle mass center</td>
</tr>
<tr>
<td>$V_{cr}$</td>
<td>Critical speed</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Tire slip angle</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Front wheel steer angle</td>
</tr>
<tr>
<td>$i$</td>
<td>Wheel index; $i=1,2,3,4$</td>
</tr>
</tbody>
</table>
6. REFERENCES


