Predicting Inflation Dynamics with Singular Spectrum Analysis

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Abstract

We use univariate and multivariate singular spectrum analysis in predicting inflation rate as well as the changes in direction of inflation time series for the United States. We use consumer price indices, and real-time chain-weighted GDP price index series in these prediction exercises. Moreover, we compare our out-of-sample h-step-ahead moving prediction results with the prediction results based on methods such as activity-based NAIRU Phillips curve, AR(p) and random walk as a naive forecasting method. We use short-run (quarterly) and long-run (one to six years) time windows for predictions and find that singular spectrum analysis outperforms all other competing prediction methods.

Keywords: Inflation rate, Prediction, Singular Spectrum Analysis.

1 Introduction

Accurate prediction of inflation rate has been a subject of great research interest for economists. The keen interest in the subject emerges from pivotally important role accurate prediction of inflation plays in macroeconomic policy analysis and decision making.

Research works on modeling and prediction of inflation began as early as 1950s. In late 1950s Phillips (1958) correlated nominal wage inflation with unemployment in the United Kingdom. Inflation studies in the United States modified this model somewhat and searched for a possible relationship between inflation and unemployment rates (Samuelson and Solow, 1960). In 1960s, Phelps (1967) and Friedman (1968) both criticized the original Phillips curve analysis by pointing out that these earlier models did not account for the effects of expectations in wage and price determination. These
latter analyses led to what is known as the accelerationist Phillips curve that assumes a relationship between the nonaccelerating inflation rate of unemployment (NAIRU) and the output gap (Stock and Watson, 1999; Gordon, 1997).

Emergence of stagflation in Europe and America in 1970s and breakdown of the inflation-unemployment nexus (Atkeson and Ohanian, 2001) motivated researchers to develop and use other methods of predicting inflation rate. These methods include application of dynamics factor models (DFMs) for construction of an index of economic activities as a proxy for unemployment rate for use in the Phillips Curve model, estimation of linear models using financial variables such as interest spreads, stock prices, money supply, among other variables (Stock and Watson, 2003) and univariate time series $AR(p)$ as well as $MA(q)$ representations of the inflation data (Gavin and Kliesen, 2006; Stock and Watson, 2005).

This study aims to predict the inflation rate using the United States’ consumer price, and chain-weighted GDP indexes. We use univariate singular spectrum analysis (SSA) and multivariate singular spectrum analysis (MSSA) in these predictions which include both the magnitude and direction of changes. Furthermore, we compare these out-of-sample predictions with those of alternative methods of inflation prediction, methods such as activity-based NAIRU Phillips curve (Atkeson and Ohanian model), $AR(p)$ and random walk as a naive forecasting method.

We use singular spectrum analysis (SSA) prediction method in this study. We are motivated to use SSA because of its ability in dealing with stationary as well as non-stationary series. Given that the dynamics of the U.S. economy has gone through many policy and structural changes during the time period under consideration, one needs to make certain that the method of prediction is not sensitive to the dynamical variations. Moreover, contrary to the traditional methods of inflation forecasting (both autoregressive or structural models that assume normality and stationarity of the series), SSA method is non-parametric model and makes no prior assumptions about the data.

Additionally, SSA method decomposes a series into its component parts, and reconstruct the series by leaving the random (noise) component behind. Furthermore, in a related, ongoing study we have discovered that the core CPI series for the United States contains a cyclical component. The data transformation of eliminating both the cyclical and random components of the time series are the main factors contributing to immense predictive power of SSA method.

In section 2 of the paper we discuss alternative methods of inflation forecasting. In section 3 we consider measures of accuracy and statistical significance of the predictions, and in Section 4 we present the empirical findings of this study. Finally, section 5 presents a summary of the study and some concluding remarks.
2 Methods of predicting inflation rate

2.1 Methods used in the previous studies

Phillips Curve and dynamic factor model

The dynamic factor model (DFMs) constructs factors (indexes) as the principal components of the set of predictors consisting of a large number of macroeconomic time series and commodity prices. The index is then used in the Phillips Curve model as a proxy for the unemployment rate. This approach has been used by Stock and Watson (1999) and Atkeson and Ohanian (2001), among others.

Specifically, Atkeson and Ohanian estimate the following model which is a modified version of NAIRU Philips curve model used by Stock and Watson (1999):

\[ \pi_{t+12}^{12} - \pi_t^{12} = \alpha + \beta(L)x_t + \gamma(L)(\pi_t - \pi_{t-1}) + \eta_{t+12} \]

where \( \pi_t^{12} \) is inflation over 12 months as measured by \( \pi_t^{12} = 100[\log(p_t) - \log(p_{t-12})] \), \( p_t \) denotes the price index in month \( t \). In model (1), \( x_t \) is the activity index constructed using dynamic factor method in conjunction with 158 or 85 monthly time series of the National Economic Activity Index (CFNAI) that is compiled by the Federal Reserve Bank of Chicago. Finally, \( \beta(L) \) and \( \gamma(L) \) are polynomials in the lag operator \( L \), and \( \eta_{t+12} \) are the error terms and are assumed to be an iid series.

Note that the left hand side of (1) is the difference between the inflation rate of next 12 months and the inflation rate of the last 12 months. Moreover, by letting \( \alpha = \beta(L) = \gamma(L) = 0 \), we can use (1) as a random walk process to conduct naive forecasts.

We use the prediction results based on Atkeson-Ohanian model which is the NAIRU-based Phillips curve in this comparative analysis\(^1\).

Autoregressive model

Another approach in inflation forecasting that appears in the literature is modeling the price indexes as \( AR(p) \) processes. In this modeling approach Akaike Information or other information criterion in determining the lag order \( p \) is often used.

2.2 Singular spectrum analysis

The SSA method is very useful in prediction of non-stationary time series. The method is highly adept in determining the principal features of a non-stationary time series process because it uses density functions derived from the singular value decomposition (SVD) of the singular vectors to generate moments that are associated with the principal features of the non-stationary process. The orthonormality of the SVD singular vectors are discrete density functions. Moments generated from these density functions are the principal features of the price indexes as nonstationary time series.

\(^1\)We are grateful to Dr. William Gavin of the Federal Reserve Bank of St. Louis for the generous supply of his inflation prediction data and the predicted data based on Atkeson and Ohanian.
It should be noted that in the SSA many probabilistic and statistical concepts are employed, however, as was stated earlier, the technique is non-parametric and does not make any statistical assumptions such as stationarity concerning either signal or noise in the data. One may consider this as one of the advantages of the technique compared to other classical methods which usually rely on some restricted assumptions such as normality or stationarity of the series (for more information see Golyandina et al. (2001)).

The traditional methods for analysis and forecasting time series such as ARMA models are restricted on the structural assumptions. For example, in order to optimally fit an ARMA model, the data must be stationary and normally distributed. These requirements do not exist for SSA, as it does not depend on any parametric model for the trend or oscillations, and does not make any assumptions about the signal or the noise component of the data.

In short, singular spectrum analysis (or in its multivariate version MSSA) decomposes a time series into its components of trend, cyclical and seasonal variations, as well as noise. Then leaving the noise aside it reconstructs the decomposed series for prediction or for identifying structural break or change point in the series.

In recent years SSA has been developed and applied to many practical problems (see, for example, Lisi and Medio, 1997; Ghil et al., 2002, Moskvina and Zhigljavsky, 2003, and Hassani, 2007). A thorough description of the theoretical foundations and practical applications of the SSA technique can be found in Golyandina et al. (2001) and Danilov and Zhigljavsky (1997).

Consider the real-valued nonzero time series \( Y_T = (y_1, \ldots, y_T) \) of sufficient length \( T \). Let \( K = T - L + 1 \), where \( L \) \((L \leq T/2)\) is some integer called the window length.

Define the matrix

\[
X = (x_{ij})_{i,j=1}^{L,K} = \begin{pmatrix}
    y_1  & y_2  & y_3 & \cdots & y_K \\
    y_2  & y_3  & y_4 & \cdots & y_{K+1} \\
    \vdots & \vdots & \vdots & \ddots & \vdots \\
    y_L  & y_{L+1} & y_{L+2} & \cdots & y_T
\end{pmatrix}
\]

and call it the trajectory matrix. Note that \( x_{ij} = y_{i+j-1} \) so that the matrix \( X \) has equal elements on the diagonals \( i + j = \text{const} \).

We then consider \( X \) as a multivariate data with \( L \) characteristics and \( K = T - L + 1 \) observations. The columns \( X_j \) of \( X \), considered as vectors, lie in an \( L \)-dimensional space \( \mathbb{R}^L \). Define the matrix \( XX^T \). Singular value decomposition (SVD) of \( XX^T \) provides us with the collections of \( L \) eigen-values \( \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_L \geq 0 \) and the corresponding eigen-vectors \( P_1, P_2, \ldots, P_L \), where \( P_1 \) is the normalized eigen-vector corresponding to the eigenvalue \( \lambda_i \) \((i = 1, \ldots, L)\). Note that one can apply SVD to a variety of matrices; for example, in addition to \( XX^T \), it is customary to use either the covariance or correlation matrix computed from \( X \), treated as a multivariate data matrix.

A group of \( r \) \((1 \leq r < L)\) eigen-vectors determine an \( r \)-dimensional hyper-plane in the \( L \)-dimensional space \( \mathbb{R}^L \) of vectors \( X_j \). The distance between vectors \( X_j \) \((j = 1, \ldots, K)\) and this \( r \)-dimensional hyper-plane can be rather small (it is controlled by the choice of the eigenvalues) meaning that the projection of \( X \) into this hyper-plane approximates well the original matrix \( X \). If we choose the first \( r \) eigen-vectors
$P_1, \ldots, P_r$, then the squared $L_2$-distance between this projection and $X$ is equal to
\[ \sum_{j=r+1}^{L} \lambda_j. \]
According to the Basic SSA algorithm, the $L$-dimensional data is projected onto this $r$-dimensional subspace and the subsequent averaging over the diagonals allows us to obtain an approximation to the original series.

Selection of the window length, $L$, which in theory of nonlinear dynamics is referred to embedding dimension, is a topic of up most importance in state space reconstruction of observed time series. Such state space reconstruction is required for an understanding of the underlying dynamics of the observed scalar series. However, a discussion of this topic is beyond the scope of the present work. We refer the interested reader to Soofi and Cao (2002) for detail discussions of window length (time delay and embedding dimension in jargon of nonlinear dynamical systems theory) selection. Nevertheless, theory of singular spectrum indicates that the window length $L \leq T/2$. However, given the superior predictive performance of the SSA relative to the competing methods, the arbitrary choice is of no practical consequence. Therefore, for brevity sake, we do not apply the usual procedures of determination of time delay and embedding dimension selection in the present study.

The use of MSSA for multivariate time series was proposed theoretically, in the context of nonlinear dynamics, by Broomhead and King (1986). There are numerous examples of successful application of the multivariate SSA (see, for example, Plaut and Vautard, 1994; Danilov and Zhigljavsky, 1997). Multivariate (or multichannel) SSA is an extension of the standard SSA to the case of multivariate time series. We give a short description of MSSA method as follows.

Assume that we have an $M$-variate time series $y_j = (y_j^{(1)}, \ldots, y_j^{(M)})$, where $j = 1, \ldots, T$ and let $L$ be window length. Using (2), we can define the trajectory matrices $X^{(i)}$ ($i = 1, \ldots, M$) of the one-dimensional time series $\{y_j^{(i)}\}$ ($i = 1, \ldots, M$). The trajectory matrix $X$ can then be defined as

\[ X = \left( \begin{array}{c} X^{(1)} \\ \vdots \\ X^{(M)} \end{array} \right). \]  

(3)

The structure of $LM \times LM$ matrix $C = XX^T$ is as follows:

\[ C = \begin{pmatrix}
C_{11} & \cdots & C_{1m} & \cdots & C_{1M} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
C_{m1} & \cdots & C_{mm} & \cdots & C_{mM} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
C_{M1} & \cdots & C_{MM} & \cdots & C_{MM}
\end{pmatrix}, \]  

(4)

where, $C_{I,J} = X^{(I)}(X^{(J)})^T$ ($I, J = 1, \ldots, M$) is an estimate of the covariance between two trajectories $X^{(I)}$ and $X^{(J)}$ corresponding to the series $Y^I$ and $Y^J$. The other stages of the Multivariate SSA procedure are identical to the Basic SSA as described above with an obvious modification that the diagonal averaging should be applied to each of the $M$ components separately.
Forecasting by SSA can be applied to the time series that approximately satisfy linear recurrent formulae (LRF):

\[ y_{i+d} = \sum_{k=1}^{d} a_k y_{i+d-k}, \quad 1 \leq i \leq T - d \]

of some dimension \( d \) with the coefficients \( a_1, \ldots, a_d \). An important property of the SSA decomposition is that, if the original time series \( Y_T \) satisfies a LRF (5), then for any \( T \) and \( L \) there are at most \( d \) nonzero singular values in the SVD of the trajectory matrix \( \mathbf{X} \); therefore, even if the window length \( L \) and \( K = T - L + 1 \) are larger than \( d \), we only need at most \( d \) matrices \( \mathbf{X}_i \) to reconstruct the series.

SSA forecasting algorithm is based on a premise which, roughly speaking, states that: If the number of terms \( r \) in the SVD of the trajectory matrix \( \mathbf{X} \) is smaller than the window length \( L \), then the series satisfies some LRF of some dimension \( d \leq r \). Let us formally describe the forecasting algorithm under consideration (for more information see Golyandina et al. (2001)):

**Algorithm input:**
(a) Time series \( Y_T = (y_1, \ldots, y_T) \).
(b) Window length \( L, 1 < L < T \).
(c) Linear Space \( \mathcal{L}_r \subset \mathbb{R}^L \) of dimension \( r < L \). It is assumed that \( e_L \notin \mathcal{L}_r \), where \( e_L = (0, 0, \ldots, 1) \in \mathbb{R}^L \).
(d) Number \( M \) of points to forecast.

**Notations and comments:**
(a) \( \mathbf{X} = [X_1, \ldots, X_K] \) is the trajectory matrix of the time series \( Y_T \).
(b) \( P_1, \ldots, P_r \) is an orthonormal basis in \( \mathcal{L}_r \).
(c) \( \tilde{\mathbf{X}} = [\tilde{X}_1 : \ldots : \tilde{X}_K] = \sum_{i=1}^{r} P_i P_i^T \mathbf{X} \). The vector \( \tilde{X}_i \) is the orthogonal projection of \( X_i \) onto the space \( \mathcal{L}_r \).
(d) \( \tilde{\mathbf{X}} = \mathcal{H} \mathbf{X} = [\tilde{X}_1 : \ldots : \tilde{X}_K] \) is the result of the Hankelization of the matrix \( \tilde{\mathbf{X}} \).
(e) For any vector \( Y \in \mathbb{R}^L \) we denote by \( Y_\Delta \in \mathbb{R}^{L-1} \) the vector consisting of the last \( L - 1 \) components of the vector \( Y \), while \( Y^\nabla \in \mathbb{R}^{L-1} \) is the vector of the first \( L - 1 \) components of the vector \( Y \).
(f) We set \( v^2 = \pi_1^2 + \ldots + \pi_r^2 \), where \( \pi_i \) is the last component of the vector \( P_i \) \( (i = 1, \ldots, r) \).
(g) Suppose that \( e_L \notin \mathcal{L}_r \) (This implies that \( \mathcal{L}_r \) is not a vertical space). Then \( v^2 < 1 \). It can be proved that the last component \( y_L \) of any vector \( Y = (y_1, \ldots, y_L)^T \in \mathcal{L}_r \) is a linear combination of the first components \( (y_1, \ldots, y_{L-1}) \):

\[ y_L = a_1 y_{L-1} + \cdots + a_{L-1} y_1. \]

Vector \( A = (a_1, \ldots, a_{L-1}) \) can be expressed as

\[ A = \frac{1}{1 - v^2} \sum_{i=1}^{r} \pi_i P_i^\nabla \]
and does not depend on the choice of a basis \( P_1, \ldots, P_r \) in the linear space \( \mathcal{L}_r \). In the above notations, define the time series \( \tilde{Y}_{T+M} = (y_1, \ldots, y_{T+M}) \) by the formula

\[
\tilde{y}_i = \begin{cases} 
\tilde{y}_i & \text{for } i = 1, \ldots, T \\
\sum_{j=1}^{T-1} a_j y_{i-j} & \text{for } i = T + 1, \ldots, T + M
\end{cases}
\]

The numbers \( y_{T+1}, \ldots, y_{T+M} \) form the \( M \) terms of the SSA recurrent forecast.

3 Measures of accuracy and statistical significance of the predictions

In this study, to measure the performance of the methods of prediction of inflation rate we use the root mean square error. To make sure that our results are not chance occurrence, we use modified Diebold-Mariano test statistics discussed in Harvey et al. (1997). Additionally, we use the direction of change criterion which shows the proportion of forecasts that correctly predict the direction of the movement of the series.

Root mean square of errors (RMSE)

As a measure of prediction accuracy, we use the following ratio of root-mean-square errors (RMSE):

\[
\text{RMSE} = \left( \frac{\sum_{i=1}^{n} (y_{T+i} - \tilde{y}_{T+i})^2}{\sum_{i=1}^{n} (y_{T+i} - \tilde{y}_{T+i})^2} \right)^{1/2}.
\]

Here \( n \) represents the number of forecasted points, \( \tilde{y}_{T+i} \) are the forecasted values of \( y_{T+i} \) obtained by SSA (or MSSA) and \( \tilde{y}_{T+i} \) is the forecasted values of \( y_{T+i} \) obtained by other method. Note that \( \tilde{y}_{T+i} \) for Random walk (RW) model is \( y_{T+i-h} \) for any \( h \)-step ahead forecasting. If \( \text{RMSE} < 1 \), then SSA (or MSSA) procedure outperforms alternative prediction method. Alternatively, \( \text{RMSE} > 1 \) would indicate that the performance of the corresponding SSA procedure is worse than the predictions of the competing method.

For the quarterly predictions of GDP price index we have only single observations. Accordingly we use the RMSEs are based on \( n = 1 \).

Diebold-Mariano significance test

As stated above, to check if the differences between the two forecasting procedures are statistically significant we applied the Diebold and Mariano (1995) test statistic with the corrections suggested by Harvey et al. (1997). The quality of a forecast is to be judged on some specified function \( g(\varepsilon) \) as a loss function of the forecast error, \( \varepsilon \). Then, the null hypothesis of equality of expected forecast performance is \( E(d_t) = 0 \), where \( d_t = [g(\varepsilon_{\text{SSA}}) - g(\varepsilon_{\text{RW}})] \) and \( \varepsilon_{\text{SSA}} \) and \( \varepsilon_{\text{RW}} \) are the forecast errors obtained with SSA and RW model, or the other methods, respectively. In our case, \( g \) is the quadratic loss function. The Diebold and Mariano statistic for \( h \) step ahead forecast and the number
of \( n \) forecasted points is

\[
S = \bar{d} \sqrt{\frac{n + 1 - 2h + h(h - 1)/n}{n \, \text{var}(\bar{d})}}
\]

where \( \bar{d} \) is the sample mean of the \( d_t \) series and \( \text{var}(\bar{d}) \) is, asymptotically

\[
n^{-1} \left( \hat{\gamma}_0 + 2 \sum_{k=1}^{h-1} \hat{\gamma}_k \right),
\]

where \( \hat{\gamma}_k \) is the \( k \)-th autocovariance of \( d_t \) and can be estimated by

\[
n^{-1} \sum_{t=k+1}^{n} (d_t - \bar{d})(d_{t-k} - \bar{d}).
\]

The \( S \) statistic follows the asymptotic standard normal distribution under the null hypothesis and its correction for finite sample follows the Student’s \( t \) distribution with \( n - 1 \) degrees of freedom.

### Direction of change criterion

The third characteristic computed for each method is the direction of change criterion (DC). It shows the proportion of forecasts that correctly predict the direction of the series movement. Let \( Z_t \) \((t = T + 1, \ldots, T + n)\) takes a value 1 if the forecast series correctly predicts the direction of change and 0 otherwise. The Moivre-Laplace central limit theorem implies that for large samples the test statistic \( 2(\bar{Z} - 0.5)n^{1/2} \) is approximately distributed as standard normal. When \( \bar{Z} \) is significantly larger than 0.5, the forecast is said to have the ability to predict the direction of change. Alternatively, if \( \bar{Z} \) is significantly smaller than 0.5, the forecast tends to give the wrong direction of change.

### 4 The empirical results

The data

We use several U.S. price indexes in out-of-sample, h-step-ahead moving prediction exercises. These indexes including consumer price index with and without highly volatile food and energy items, CPI-all and CPI-core, respectively, as well as real-time quarterly chain-weighted GDP price index. Specifically, we used monthly CPI-all and CPI-core data for the period JAN 1986 - DEC 2006. The real-time chain-weighted GDP price series consists of observations starting in the first quarter of 1959 and ending in the third quarter of 1999.


We use moving h-step-ahead prediction, which means that we include all available information for the predictions. This means that for 1-step-ahead prediction, after using \( y_1 \cdots y_t \) in prediction of \( y_{T+1} \), we use all observations \( y_1 \cdots y_{T+1} \) in prediction of \( y_{T+2} \), and so forth.

In addition to using real-time Chain-weighted GDP price index, we also used the GNP/GDP deflater and we find our prediction results based on these two data sets are
very similar. Given higher volatility of inflation during 1980s, we report the prediction results based on GNP/GDP revised data in Table 2 below also. Based on this observation, we only use published data on CPI to make our prediction comparable with inflation rate prediction based on CPI by other researchers who used published CPI data also.

Forecasting Inflation rate based on the CPI-all and CPI-core series

Next, we present the forecasting results for inflation rate based on the Consumer Price Indices for the long and short horizons. We use MSSA for forecasting inflation rate based on the CPI-all and CPI-core series over the period Jan-1986 to Dec-1996 that was used as the training set data.

To perform SSA, first we need to choose the window length \( L \) (which is the only parameter in the decomposition stage). Selection of the proper window length depends on preliminary information about the time series. Theoretical results tell us that \( L \) should be large enough but not greater than \( T/2 \). Furthermore, if we know that the time series may have a periodic component with an integer period (for example, if this component is a seasonal component), then to get better separability of this periodic component it is advisable to take the window length proportional to that period. Based on these considerations, we take \( L = 60 \). The length of CPI-core series is \( T = 132 \). Therefore, based on this window length and considering the SVD of the matrix \( XX^T \), we have 60 eigentriples, which are ordered by their contribution (shares) in the decompositions, as well as 60 eigenvectors and principal components.

We select the window length \( L = 60 \) and the first 12 eigenvalues for reconstructing the original series without noise and consider remaining eigentriples (13-60) as noise (for more information about proper selection of eigenvalues see Golyandina et al. (2001)).

Short horizon forecast: 1 and 3-step ahead forecast

Table 1 shows the RMSEs for MSSA/random walk for 1-step and 3-step ahead forecasts of inflation rate based on the CPI-all and CPI-core series for a number of periods. The results indicate that MSSA outperforms the random walk predictions in both one and 3-step ahead forecasts and in all time periods considered in the table. Table 1 also presents the results of Diebold and Mariano test indicating whether the discrepancies between MSSA and RW model forecasting procedures are statistically significant. The results of this table confirm that, for all cases, the differences are significant at 1% confidence level.

Additionally, Table 1 presents test results for the null hypothesis of whether the percentages of the direction of changes are greater than the pure chance (50%). The table shows that all results are statistically significant at 1% confidence level. It should be noted that the MSSA prediction results for CPI-core series are better than MSSA prediction results for the CPI-all series. The results of this table also show that MSSA predicts direction of change for 3-step as accurately as it can predict 1-step ahead.
Table 1: RMSE of MSSA forecast results with respect to the RW method, Diebold-Mariano significance test results and direction of change test for inflation rate based on the CPI-all and CPI-core series. ** and * imply significance at 1% and 10% confidence levels, respectively.

Next we consider the time profile of predicted and the actual values by comparing 1-step and 3-step ahead predictions by MSSA for inflation prediction based on the CPI-core series, by considering Fig. 1. Clearly, the MSSA forecast series and the inflation rate move in very close proximity of each other.

To acquire a better understanding of forecasting accuracy of the methods, we examine the empirical cumulative distribution function for the absolute errors of the MSSA and RW methods next. Fig. 2 presents the empirical cumulative distribution function (CDF) for the absolute errors of the MSSA and RW forecasts. The graph on the left is for 1-step ahead and to the right is 3-step ahead predictions.

Fig. 2 shows that the empirical distribution of the RW errors stochastically dominates the distribution of the MSSA errors (that is, the RW errors are stochastically larger than the MSSA errors). It appears from Fig. 2 that the frequencies of larger errors for the random walk model are substantially higher compared to MSSA’s errors. In fact, the maximum error for MSSA in both 1-step and 3-step prediction is 0.003, while the maximum error for the random walk is almost 0.008 for 3-step ahead prediction, and the maximum error for 1-step ahead is approximately 0.006.

Figure 1: MSSA predicted results of inflation rate series based on the CPI-core series (thick line) and the inflation rate series (thin line) for 1-step ahead forecast (left side) and 3-step ahead forecast (right side) over the period JAN 1997 to DEC 2006.
Figure 2: Empirical cumulative distribution functions of the absolute errors for MSSA (thick line) and random walk (dashed line) for 1-step ahead (left side) and 3-step ahead forecast (right side) over the period JAN 1997 to DEC 2006.

Long horizon forecast: 120–step–ahead forecast

Let us now consider MSSA forecast for 120–step–ahead forecast of CPI-all and CPI-core series. That is, we use these series indicated (two CPI data sets for \( h = 1 \) and \( h = 3 \)) over period JAN 1986-DEC 1996 to reconstruct noise–free series (the training set) and then use the refined series to forecast 120 data points over period JAN 1997–AUG 2006 (the out-of-sample prediction set). Here we compare 120–step–ahead MSSA forecast results with 1 and 3-step ahead RW forecast. That is, we update information, step by step, for RW model only, but do not update information for MSSA.

Table 2 shows the RMSEs for 120–step–ahead forecast for MSSA, MSSA\(_{120}\), and 1-step and 3-step ahead RW forecasts, RW\(_1\) and RW\(_3\), of inflation rate based on the CPI-all and CPI-Core series for a number of periods. The results indicate that MSSA outperforms the random walk predictions in all time periods considered in the table. Diebold and Mariano tests also confirm that the discrepancy between MSSA and RW model forecasting inflation rate based on the CPI-core series are statistically significant at 1% confidence level.

The results of direction of change test statistics are also presented in the Table 2. The table shows that all results are statistically significant at 1% confidence level with the exception of 3 cases that are significant at 5% confidence level. Therefore we conclude that the MSSA method can predict direction of change for long horizons forecast as well as short horizon forecast even for 120-step ahead forecast.

Comparison with the other methods

Comparative study is somewhat difficult, since data, methods, forecasting horizons and error criteria are not uniform. Here we compare the results based on the MSSA method with the results obtained from other inflation prediction methods. We smoothed the series by taking 3-month moving averages to make our results are comparable with prediction results of other inflation researchers who had smoothed the CPI data by the moving average method (Gavin and Kliesen, 2007). Table 3 presents RMSEs of MSSA prediction results and the forecasting results obtained using the other models.
<table>
<thead>
<tr>
<th>Year</th>
<th>N</th>
<th>RMSE (MSSA\textsubscript{120}/RW\textsubscript{3})</th>
<th>Direction of Change</th>
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<td></td>
<td></td>
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<td>h=3</td>
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<td></td>
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</tr>
<tr>
<td>Jan 97-Dec 02</td>
<td>72</td>
<td>0.76*</td>
<td>0.65***</td>
</tr>
<tr>
<td>Jan 97-Dec 04</td>
<td>96</td>
<td>0.82*</td>
<td>0.67***</td>
</tr>
<tr>
<td>Jan 97-Dec 06</td>
<td>120</td>
<td>0.86*</td>
<td>0.74***</td>
</tr>
</tbody>
</table>

Table 2: RMSE of 120-step ahead forecast for MSSA, MSSA\textsubscript{120} with respect to the RW method for 1-step ahead RW\textsubscript{1} and 3-step ahead forecast RW\textsubscript{3}. Diebold-Mariano significance test results: ***, ** and * imply significant at 1%, 5% and 10% confidence level, respectively.

considered in this study for 3-step ahead predictions\textsuperscript{2}. It should be noted that due to minor differences between our CPI data distributed by the United States Bureau of Labor Statistics, and the CPI data used by other researchers that were apparently obtained from a commercial entity, we calculate the RMSE's by the ratio of [SSA/RW (our data)]/ [Other model/RW (their data)].

It is instructive to note that the Fed has targeted inflation rate it aims to steer the economy toward. The targeted inflation rates by the Fed tend to be rather similar to the predicted results by Phillips curve, DFM and AR(p) models, implying that the outcomes of these latter models are somewhat similar to the intuition of policy makers about inflationary pressure in the economy. Perhaps, the Fed’s targeted inflation rates are based on the predictions of these forecasting methods.

<table>
<thead>
<tr>
<th>Year</th>
<th>N</th>
<th>MSSA/AO</th>
<th>MSSA/AR</th>
<th>MSSA/DF\textsubscript{88}</th>
<th>MSSA/DFM\textsubscript{88}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>CPI-all</td>
<td>CPI-core</td>
<td>CPI-all</td>
<td>CPI-core</td>
</tr>
<tr>
<td>Jan 97-Dec 98</td>
<td>24</td>
<td>0.54</td>
<td>0.34</td>
<td>0.57</td>
<td>0.27</td>
</tr>
<tr>
<td>Jan 97-Dec 00</td>
<td>48</td>
<td>0.73</td>
<td>0.37</td>
<td>0.79</td>
<td>0.31</td>
</tr>
<tr>
<td>Jan 97-Dec 02</td>
<td>72</td>
<td>0.68</td>
<td>0.45</td>
<td>0.73</td>
<td>0.39</td>
</tr>
<tr>
<td>Jan 97-Dec 04</td>
<td>96</td>
<td>0.70</td>
<td>0.42</td>
<td>0.70</td>
<td>0.40</td>
</tr>
<tr>
<td>Jan 97-Aug 06</td>
<td>118</td>
<td>0.75</td>
<td>0.39</td>
<td>0.76</td>
<td>0.36</td>
</tr>
</tbody>
</table>

Table 3: RMSE of MSSA forecast with other models for 3-month ahead forecast for 3-month moving averages of inflation rate based on the CPI-all and CPI-core series.

In Table 4, we present results for the direction of change in the moving average series according to all inflation forecasting methods discussed in this study. The numbers in the data show the percentage of time a method correctly predicted the direction of change in a series. The numbers indicate that MSSA method correctly predicts the direction of the change of the moving average of CPI-Core consistently higher than the competing models. This is particularly true for longer prediction horizons. For example, compare number 0.84 under MSSA-CPI-Core column for period of January 1997–August 2006, with the remaining entries in the same row. The superior performance of MSSA for CPI-all for period equal to or longer than 96 observations is

\textsuperscript{2}The labor intensive work on predicting 12-month and 24-month ahead predictions are underway, and we hope to present these results in future in another paper.
apparent also. The statistical significance of the predicted values are also presented in the table.

<table>
<thead>
<tr>
<th>Year</th>
<th>N</th>
<th>MSSA</th>
<th>AO</th>
<th>AR</th>
<th>DFM88</th>
<th>DFM108</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>CPIall</td>
<td>CPIcore</td>
<td>CPIall</td>
<td>CPIcore</td>
<td>CPIall</td>
</tr>
<tr>
<td>Jan 97-Dec 98</td>
<td>24</td>
<td>0.87**</td>
<td>0.96**</td>
<td>0.89**</td>
<td>0.89**</td>
<td>0.91**</td>
</tr>
<tr>
<td>Jan 97-Dec 00</td>
<td>48</td>
<td>0.73**</td>
<td>0.96**</td>
<td>0.73**</td>
<td>0.76**</td>
<td>0.80**</td>
</tr>
<tr>
<td>Jan 97-Dec 02</td>
<td>72</td>
<td>0.70**</td>
<td>0.86**</td>
<td>0.57**</td>
<td>0.62**</td>
<td>0.72**</td>
</tr>
<tr>
<td>Jan 97-Dec 04</td>
<td>96</td>
<td>0.71**</td>
<td>0.88**</td>
<td>0.43</td>
<td>0.50</td>
<td>0.56</td>
</tr>
<tr>
<td>Jan 97-AUG 06</td>
<td>116</td>
<td>0.72**</td>
<td>0.84**</td>
<td>0.28</td>
<td>0.37</td>
<td>0.44</td>
</tr>
</tbody>
</table>

Table 4: Direction of change results of 3-month ahead forecasts of the moving average series. * and ** indicate the 10% and 1% levels of significance, respectively.

**Inflation rate based on the GDP price index**

Based on SSA decomposition of GDP deflater, we notice a cyclical component in the series. Accordingly, based on trial and error we selected an optimal window length.

In the reconstruction stage of SSA, we let $L = 3$ and choose 2 eigentriples for forecasting inflation rate based on the GDP price index over period 1992.Q1 to 1998.Q4. We use the RW model as a benchmark model in the comparative analyses. The use of the random walk as a benchmark model is motivated by a paper by Stock and Watson (2005) showing that a simple random walk model appears to work as well as more complicated models in prediction of inflation. Given the superior performance of the random walk model compared to the Green Book predictions, as documented in the Stock and Watson (2005) we do not directly compare SSA forecast with GB forecast. The use of the random walk model as a benchmark model should not imply that we believe the model is an optimal forecasting method. We use this model because it is a naïve model. The point here is that a superior performance of random walk model would render the analyst’s method useless.

Table 5 shows the annual predictions of the GDP price index and the inflation rates based on SSA-random walk (SSA/RW). The quarterly data that appear in columns 2–5 show the RMSEs calculated with one observation. The annual data that appear in column 6 show RMSE for each year which is obtained by using data from four quarters. The last two columns of this table show the results of inflation predictions. The column labeled period indicates the time interval for forecasting and comparisons. For example, period 92–98, shows the RMSE for period 1992 to 1998. Note that the data in the last two columns of the table are RMSEs for one to 6 years predictions.

Without exception, SSA outperforms the naïve random walk method. The most efficient SSA performance over the random walk method goes up to 41% for 1992 prediction. From the results of Table 5 (the last column), it appears that prediction of the inflation rate (first differenced log of GDP price index) is more challenging than prediction of GDP price index.
<table>
<thead>
<tr>
<th>Year</th>
<th>Quarterly GDP Price Index</th>
<th>Annual GDP price Index</th>
<th>Period</th>
<th>Inflation Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Q1</td>
<td>Q2</td>
<td>Q3</td>
<td>Q4</td>
</tr>
<tr>
<td>1977</td>
<td>0.07</td>
<td>0.46</td>
<td>0.50</td>
<td>0.63</td>
</tr>
<tr>
<td>1978</td>
<td>0.40</td>
<td>0.17</td>
<td>0.14</td>
<td>0.40</td>
</tr>
<tr>
<td>1979</td>
<td>0.29</td>
<td>0.44</td>
<td>0.05</td>
<td>0.44</td>
</tr>
<tr>
<td>1980</td>
<td>0.14</td>
<td>0.25</td>
<td>0.17</td>
<td>0.42</td>
</tr>
<tr>
<td>1981</td>
<td>0.13</td>
<td>0.70</td>
<td>0.50</td>
<td>0.17</td>
</tr>
<tr>
<td>1982</td>
<td>0.57</td>
<td>0.55</td>
<td>0.36</td>
<td>0.45</td>
</tr>
<tr>
<td>1983</td>
<td>0.71</td>
<td>0.47</td>
<td>0.55</td>
<td>0.35</td>
</tr>
<tr>
<td>1984</td>
<td>0.54</td>
<td>0.36</td>
<td>0.58</td>
<td>0.46</td>
</tr>
<tr>
<td>1985</td>
<td>0.64</td>
<td>0.68</td>
<td>0.98</td>
<td>0.61</td>
</tr>
<tr>
<td>1986</td>
<td>0.21</td>
<td>0.29</td>
<td>0.45</td>
<td>0.45</td>
</tr>
<tr>
<td>1987</td>
<td>0.54</td>
<td>0.56</td>
<td>0.30</td>
<td>0.40</td>
</tr>
<tr>
<td>1988</td>
<td>0.37</td>
<td>0.44</td>
<td>0.43</td>
<td>0.61</td>
</tr>
<tr>
<td>1989</td>
<td>0.42</td>
<td>0.24</td>
<td>0.70</td>
<td>0.40</td>
</tr>
<tr>
<td>1990</td>
<td>0.67</td>
<td>0.42</td>
<td>0.60</td>
<td>0.57</td>
</tr>
<tr>
<td>1991</td>
<td>0.57</td>
<td>0.70</td>
<td>0.55</td>
<td>0.42</td>
</tr>
<tr>
<td>1992</td>
<td>0.07</td>
<td>0.21</td>
<td>0.50</td>
<td>0.32</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SSA/RW: GDP Deflator</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.16</td>
</tr>
<tr>
<td>0.24</td>
</tr>
<tr>
<td>0.11</td>
</tr>
<tr>
<td>0.32</td>
</tr>
<tr>
<td>0.03</td>
</tr>
<tr>
<td>0.26</td>
</tr>
<tr>
<td>0.88</td>
</tr>
</tbody>
</table>

Table 5: The root mean squared error (RMSE) of SSA-random walk for the quarterly and annual real-time GDP chain-weighted price index and inflation rate forecasts: the United States, 1992.I to 1998.IV.

Note: The data under Quarterly GDP Price Index, columns 2–5 are RMSEs calculated with one observation.

5 Summary and conclusion

We demonstrated that the SSA technique is useful in predicting the economic time series. We used several price indexes including consumer price index with and without highly volatile food and energy items as well as quarterly Chain-weighted GDP price index for forecasting inflation rate.

The results of this paper show that the SSA significantly outperforms all other methods commonly used in inflation forecasting. We believe our superior prediction results are based on the ability of the SSA method to discard the stochastic components of the original series. In a related study, we have identified clear structures in the price series and see that SSA is able to clearly identify and discard these structures that tend to increase the volatility of the series.

The results show that without exception, SSA outperforms both the naïve random walk method and highly complex econometric model that is used by the Federal Reserve System for forecasting inflation rate based on the GDP price index (see Table 2). Moreover, we find that MSSA outperforms the random walk predictions in both one and 3-step ahead forecasts as well as all other time periods considered for forecasting inflation rate based on the CPI-all and CPI-core series (see Table 1). The Diebold and Mariano tests also confirm that, for all cases, the results are significant at 1%
confidence level. We also find that SSA performs very well in predicting the direction of change. Additionally, we find that the empirical distribution of the RW errors also stochastically dominates the distribution of the MSSA errors for one and 3-step ahead forecast (see Fig. 2).

We also consider MSSA forecast for 120-step-ahead forecast. We compared 120-step-ahead MSSA forecast results with 1 and 3-step ahead RW forecast. The results indicate that MSSA outperforms the random walk predictions in all time periods considered (see Table 2).

Diebold and Mariano tests also confirm that the discrepancy between MSSA and RW model forecasting procedures are statistically significant at 1% confidence level. The results indicate that MSSA method can predict direction of change for long horizon forecast as well as short horizon forecast even for 120-step-ahead predictions.

We also compared the MSSA forecasting results with those results obtained by Phillips curve, DFM and AR(p) models. Once again, MSSA outperforms all other models for forecasting inflation rate and direction of change in the CPI-all and CPI-core (see Tables 3 and 4).

Finally, in light of inadequate performances of the NAIRU Philips curve and the time series models, we conclude that using SSA and MSSA is more promising for obtaining accurate forecasting of inflation rate.
References


Federal Reserve Bank of Philadelphia., Greenbook Forecasts


