Each correct answer is worth 10 points.

1: Simplify $\left[1 - 3(1 - 3)^{-1}\right]^{-1}$.

2: One-fourth of a number is two more than one-fifth of the number. Find the number.

3: How many perfect cubes greater than one are divisors of $9^9$.

4: A line segment joins points $A(5, 6)$ and $C(20, 36)$. Find the coordinates of the point $B$ on this segment so that $2AB = BC$. (That is, point $B$ trisects the segment.)

5: A positive number times four more than the number is equal to forty more than the number. Find the number.

6: A track is the border of the region that is formed when two semicircular regions are adjoined to a rectangular region $50$ meters by $x$ meters. If the total area of the semicircular regions is half the area of the rectangular region, find $x$.

7: How many three digit numbers are there which have only odd digits?

8: Simplify: \[\frac{\sqrt{2}}{1 - \sqrt{2}} + \frac{\sqrt{2}}{1 + \sqrt{2}}.\]

9: Two concentric circles have radii $2$ inches and $3$ inches. If a chord of the larger circle is constructed tangent to the smaller circle, what is the length of the chord?

10: There are several cows and farmers in a barn. One of the farmers notes that there are $82$ feet and $26$ heads in the barn. How many cows are there?
Each correct answer is worth 25 points.

1. In a small town 35% of the registered voters were democrats. One year 50 new voters registered, 30 of which are democrats. Now 36% of the voters are democrats. How many registered voters are there now?

2. A plane departs from City A at 10 am and flies due east arriving at city B at 7 pm. A second plane leaves city B at noon and arrives at City A at 3 pm. If the two planes travel at the same speed, what is the difference in the time zones for A and B?

3. Find the 200th term of the sequence 418, 424, 434, 440, 450, 456, 466, . . .

4. Ramp AB is 25 feet long, point A is at ground level and point B is 7 feet above the ground. A box 3 feet long and 1.5 feet high sits at the top of the ramp. How far is point D above the ground?

5. Find the result when the base seven numbers 163 and 251 are multiplied together. Give your answer as a base seven number.

6. Define the operation $\oplus$ by $a \oplus b = a + b - 7$. The number $z$ is called the zero for the operation $\oplus$ if $a \oplus z = a = z \oplus a$ for every value of $a$. Given $a$, the number $b$ is called the inverse of $a$ if $a \oplus b = z = b \oplus a$. Find the value of $z$ and the inverse of 5.

7. Quadrilateral ABCD has vertices A(0, 10), B(6, 2), C(1, y) and D(-4, 4). If the area of ABCD is 25 square units, find the value of y.

8. There is one way to fit two regular hexagons together edge-to-edge and three distinct ways to fit three regular hexagons together edge-to-edge. How many distinct ways are there to fit four regular hexagons together edge-to-edge?
Each correct answer is worth 10 points.

_______1: If \( \theta \) is in quadrant IV and \( \sec \theta = 1 + 6 \cos \theta \), find \( \sin \theta \).

_______2: Solve for \( x \): \( 2^x - 2^{x-2} = 192 \).

_______3: If \( \frac{15}{a\sqrt{2} + b\sqrt{3}} = \sqrt{2} + \sqrt{3} \), find the values of \( a \) and \( b \).

_______4: If \( f(x) = \frac{1}{x+1} \), compute \( f(f(x+2)) \). Simplify your answer.

_______5: A square piece of paper is rolled into a cylinder of volume \( 250 \text{ cm}^3 \). What are the dimensions of the square?

_______6: How many three digit numbers are there that have at least one odd digit?

_______7: The line \( y = bx - b \) intersects the parabola \( y = x^2 \) in exactly one point. If \( b \neq 0 \), what is the value of \( b \)?

_______8: Define \( a \oplus b = a + b - 7 \). The number \( z \) is called the zero for \( \oplus \) if \( a \oplus z = a = z \oplus a \) for any value of \( a \). Given \( a \), the number \( b \) is called the inverse of \( a \) if \( a \oplus b = z = b \oplus a \). Find the value of \( z \) and the inverse of 5.

_______9: Find the length of the shortest altitude in a 3 - 4 - 5 triangle.

_______10: A square has two vertices on the parabola \( y = 8 - 2x^2 \) and two vertices on the \( x \)-axis. Find the area of the square.
Level II — Team Event

Each correct answer is worth 25 points.

1. What is the 65th term in the sequence -72, -71, -69, -66, -62, . . . ?

2. The harmonic mean of two numbers is defined to be the reciprocal of the average of the reciprocals of the two numbers. Find \( x \) so that the harmonic mean of 5 and \( x \) is 8.

3. Sam visits an art gallery and observes a large picture on a wall 20 feet away. If the bottom of the picture is two feet above Sam’s eye level and the picture subtends an angle of 30° at Sam’s eye, what is the height of the picture? Give your answer to the nearest inch.

4. What is the coefficient of \( x \) in the expansion of \((x + 3)^4 (x - 2)^7\) ?

5. A triangular array of six circles is inscribed in an equilateral triangle as shown. If the radius of each circle is one unit, find the total area of the regions that lie inside the triangle and outside of the circles.

6. Two numbers are chosen at random from 1, 2, 3, 4, . . . , 20. What is the probability that the smaller number divides into the larger?

7. Find the coordinates of the point on the circle \( x^2 + y^2 = 16 \) which is closest to the line \( y = 20 - 2x \).

8. A square spiral bug begins at the origin and travels 1000 units due north (along the positive y-axis). It then turns right and travels 900 units due east (in the positive x direction). It then turns right and travels due south 810 units, and turns right again and travels due west 729 units. If the bug were to continue this pattern indefinitely, find the coordinates of the point where it would end up.