1. The segment connecting (0, 0) to (1000, 2012) has slope $\frac{503}{250}$. Thus a lattice point is obtained only for $x = 0, 250, 500, 750$ and 1000. Hence there are 5 lattice points on the segment.

2. Let $x$ be the length of the string, then $40 = x \sin 60^\circ - x \sin 45^\circ = x \left( \sqrt{3}/2 - \sqrt{2}/2 \right)$. Thus the length of the string is $x = \frac{80}{\left(\sqrt{3} - \sqrt{2}\right)} = 80 \cdot \left(\sqrt{3} + \sqrt{2}\right)$ feet.

3. The probability that the socks match is
\[
\frac{\binom{14}{2} + \binom{8}{2} + \binom{4}{2}}{\binom{26}{2}} = \frac{\frac{14 \cdot 13}{2} + \frac{8 \cdot 7}{2} + \frac{4 \cdot 3}{2}}{\frac{26 \cdot 25}{2}} = \frac{91 + 28 + 6}{325} = \frac{125}{325} = \frac{5}{13}.
\]

4. Let $y = x^{1/6}$, then $x^{5/6} - x^{2/3} = 2x^{1/2} \Rightarrow y^5 - y^2 = 2y^3 \Rightarrow y^3(y^2 - y - 2) = 0 \Rightarrow y^3(y - 2)(y + 1) = 0$. Thus $y = 0, 2$ or -1. But $y = -1$ is extraneous, so $y = 0$ or 2 and $x = 0$ or 64.

5. $\frac{x(x-1) - (m-1)}{(x-1)(m-1)} = \frac{x}{m} \Rightarrow m \cdot (x^2 - x - m + 1) = x \cdot (xm-x-m+1) \Rightarrow x^2m-xm-m^2+m = x^2m-x^2-xm+x \Rightarrow x^2-x+m-m^2 = 0$. This last equation has a unique solution if $m^2 - 4m + 1 = 0 \Rightarrow (2m - 1)^2 = 0$. Thus $m = 1/2$.

6. The midpoint of the segment joining (1, 2) and (3, -12) is (2, -5). This segment has slope -7 and the perpendicular bisector of this segment has equation $7y - x = -37$. The midpoint of the segment joining (3, -12) and (7, 10) is (5, -1). The slope of this segment is 11/2 and the perpendicular bisector of this segment has equation $11y + 2x = -1$. The lines $7y - x = -37$ and $11y + 2x = -1$ intersect at the point (16, -3) which is the desired circumcenter.

7. There is one coloring where the squares are either all white or all black. There are two distinct colorings with either one black or one white square. There are six distinct colorings with either two black or two white squares. There are six distinct colorings with three black and three white squares. Hence there are a total of $2(1 + 2 + 6) + 6 = 24$ distinct colorings.

8. Using the information given in the problem we can determine the lengths as shown in the diagram. Thus the base of the triangle is 24 inches and, since the slope of a side of the triangle is 2, the height of the triangle is also 24 inches. Therefore the area of the triangle is $24 \cdot 24 / 2 = 288$ square inches. Now, using the summation formula for a geometric series, the total area in the squares is $144 + 36 + 9 + \ldots = \frac{144}{1 - 1/4} = 192$ square inches, and the area inside the triangle and outside of the squares is $288 - 192 = 96$ square inches.