UW-Platteville Mathematics Meet – 2011

Level II — Team Event

Each correct answer is worth 25 points.

1. Let $x$ and $b$ be positive integers. If $x$ is represented by 324 in base $b$, and $x$ is represented as 155 in base $b + 2$, what is $b$?

2. What is the units digit of $6^{25} - 3^{48}$?

3. In the diagram at right, a line through point C is constructed so that $\angle ACE \equiv \angle DCE$. Find the coordinates of point $E$.
   
   Note: $AC$ is parallel to $OD$.

4. A three-digit number, $abc$, is in arithmetic progression if $b = a + k$ and $c = b + k$ for some integer $k$. How many three-digit numbers are in arithmetic progression?
   
   Note: The first digit can not be zero, and $k$ is not necessarily positive.

5. In the figure at right, assume that $\frac{BD}{DC} = \frac{1}{4}$, $\frac{CE}{EA} = 1$ and

   \[
   \text{area}(\triangle DEF) = \frac{9}{32} \cdot \text{area}(\triangle ABC).
   \]

   Compute $\frac{AF}{FB}$.

6. Suppose that $f(0) = 1$, $f(1) = 0$ and for any integer $n \geq 1$, $f(2n) = f(n) + 1$ and $f(2n + 1) = f(n) - 1$. Find $f(2011)$.

7. Simplify: \[
\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right) \cdots \cdots \left(1 - \frac{1}{2011^2}\right).
\]

   Note: each factor is in the form $a^2 - b^2$.

8. Let $l$ be a line through the point $(1, 0)$ which intersects the parabola $y = x^2 + 1$ in exactly one point, what is the slope of $l$?