Each correct answer is worth 25 points.

1. The polynomial $x^2 - 10x + 5$ has roots $r$ and $s$. Find the values of $b$ and $c$ so the polynomial $x^2 + bx + c$ has roots $r^2$ and $s^2$.

2. A cube and a sphere have the same surface area. What is the ratio of the volume of the sphere to the volume of the cube?

3. Find integers $A$, $B$ and $C$ so that $\frac{A}{x+2} + \frac{B}{x+5} + \frac{C}{x} = \frac{4x^2 + 33x + 20}{x^3 + 7x^2 + 10x}$.

4. For $N > 0$, simplify $\sqrt[3]{N} \sqrt[3]{N} \sqrt[3]{N} \cdots$.

5. Find the exact value of $\cos(20^\circ) \cdot \cos(40^\circ) \cdot \cos(80^\circ)$.

6. Noah’s house has a staircase with 12 steps. He can go down the steps one at a time or two at a time. For example: He could go down 1 step, then 1 step, then 2 steps, then 2, 2, 1, 1, 1 steps. In how many ways can Noah go down the 12 steps, taking one or two steps at a time?

7. A square is constructed on the hypotenuse of a $30^\circ - 60^\circ - 90^\circ$ right triangle as shown. If the length of the shorter leg of the triangle is $x$ units, compute the distance from $A$ to $D$.