1. We know that and that $3^2 + 4^2 = 5^2$ and that $5^2 + 12^2 = 13^2$, so one solution is $x = 3, y = 4, z = 12$ and $w = 13$.

2. Construct $AX$ and $AY$ perpendicular to $MN$ and $NP$, respectively. Since $\triangle AXE$ and $\triangle AFY$ have the same area, the overlapping region is one fourth of the area of both squares, so the area is $(1/4) \cdot 7^2 = 49/4$ square inches.

3. There are 14 different ways: $1 + 1 + 1 + 1 + 1 + 1 + 1$, $1 + 1 + 1 + 1 + 1 + 2$, $1 + 1 + 1 + 1 + 1 + 3$, $1 + 1 + 1 + 1 + 4$, $1 + 1 + 1 + 2 + 2$, $1 + 1 + 1 + 5$, $1 + 1 + 2 + 3$, $1 + 6$, $1 + 4 + 2$, $1 + 2 + 2 + 2$, $1 + 3 + 3$, $2 + 2 + 3$, $2 + 5$ and $3 + 4$.

4. Let $r$ be the radius of the center of the semicircular sections of lane 1 and $R$ be the radius of the center of the semicircular sections of lane 9. Then $R = r + 8$ and the length of the center of lane 9 is $200 + 2\pi R = 200 + 2\pi (r + 8) = 200 + 2\pi r + 16\pi = 400 + 16\pi$ meters.

5. We can use the Pythagorean Theorem to compute the altitude of each triangular face, which is $\sqrt{40^2 + 25^2} = \sqrt{2225} = 5\sqrt{89}$. Thus the area of the triangular face is $(1/2) \cdot 50 \cdot 5\sqrt{89} = 125\sqrt{89}$ square feet.

6. \[
\left( \frac{1 \cdot 2 \cdot 4 + 2 \cdot 4 \cdot 8 + 3 \cdot 6 \cdot 12 + \cdots}{1 \cdot 3 \cdot 9 + 2 \cdot 6 \cdot 18 + 3 \cdot 9 \cdot 27 + \cdots} \right)^{1/3} = \left( \frac{1 \cdot 2 \cdot 4}{1 \cdot 3 \cdot 9} \cdot \frac{1+2+3+\cdots}{1+2+3+\cdots} \right)^{1/3} = \left( \frac{8}{27} \right)^{1/3} = \frac{2}{3}
\]

7. Let $x$ be the price of a tootsie roll, $y$ be the price of a candy bar and $z$ be the price of a piece of gum. We have $4x + y + 10z = $1.69 and $3x + y + 7z = $1.26. Subtracting these equations we have $x + 3z = $0.43 $\Rightarrow x = $0.43 $- 3z$. Substitution into the first equation gives $4 \cdot ($0.43 $- 3z) + y + 10z = $1.69 $\Rightarrow y = 2z = $-0.03. Adding these two new equations gives $x + 3z + y - 2z = $0.43 $- 0.03 $\Rightarrow x + y + z = $0.40. The third kid pays 40 cents.

8. \[
\frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} + \cdots + \frac{1}{\sqrt{2005} + \sqrt{2006}} = \sqrt{1} - \frac{\sqrt{2}}{-1} + \sqrt{2} - \frac{\sqrt{3}}{-1} + \sqrt{3} - \frac{\sqrt{4}}{-1} + \cdots + \frac{\sqrt{2005} - \sqrt{2006}}{-1} = \sqrt{2} - 1 + \sqrt{3} - \sqrt{2} + \sqrt{4} - \sqrt{3} + \cdots + \sqrt{2006} - \sqrt{2005} = \sqrt{2006} - 1
\]