UW-Platteville Mathematics Meet – 2006

Level I — Team Event

Each correct answer is worth 25 points.

1. Find positive integers \(x, y, z\) and \(w\) so that \(x^2 + y^2 + z^2 = w^2\).

2. The vertex \(A\) of square \(ABCD\) is placed so that it coincides with the center of square \(MNPQ\) and \(ABCD\) is oriented so that \(AB\) trisects \(MN\). If the lengths of the sides of both squares is 7 inches, find the common area.

3. The integer 4 can be written in four ways as the sum of positive integers: \(1 + 1 + 1 + 1, 1 + 1 + 2, 1 + 3\) and \(2 + 2\). (Note that \(1 + 1 + 2, 1 + 2 + 1\) and \(2 + 1 + 1\) are all considered the same.) How many ways can 7 be written as the sum of positive integers?

4. The lanes of a track consist of two parallel straight sections with semicircular sections on each end. The center of lane 1 (the inner most lane) is 400 meters long; each straight section and each semicircular section is 100 meters. If the lanes are 1 meter wide, how long is the center of lane 9? (The straight sections of all lanes are 100 meters long.)

5. A pyramid has a square base measuring 50 feet on each side and a height of 40 feet. Find the area of each triangular face of the pyramid.

6. Evaluate \(\left(\frac{1 \cdot 2 \cdot 4 + 2 \cdot 4 \cdot 8 + 3 \cdot 6 \cdot 12 + \cdots}{1 \cdot 3 \cdot 9 + 2 \cdot 6 \cdot 18 + 3 \cdot 9 \cdot 27 + \cdots}\right)^{1/3}\).

7. Three kids went into a candy store. The first bought four tootsie rolls, a candy bar and ten pieces of gum for $1.69. The second purchased three tootsie rolls, a candy bar and seven pieces of gum for $1.26. How much did the third pay for a tootsie roll, a candy bar and a piece of gum?

8. Evaluate the sum:
\[
\frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} + \frac{1}{\sqrt{2005} + \sqrt{2006}}.
\]
Hint: Rationalize the denominators.